Ground States of Magnetic Spherical Shells

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I. INTRODUCTION

A competition between short-range exchange and long-range magnetostatic interactions results in nontrivial distribution of magnetization in a magnetic nanoparticle [1]. Such magnetic distributions as vortex and onion are widely studied for the case of planar structures. Recently a significant interest to nanomagnets with curved shape appears. On the one hand it is due to modern techniques of fabrication of nanoparticles of a custom shape, and on the other hand the curvature might significantly change magnetic properties of the sample and cause new effects [2].

The equilibrium states of magnetic nanoparticles are widely studied for different geometries: discs, rings, tubes, extruded hemispheres and caps. It has been shown that magnetic nanoparticle of ring shape has three possible ground states: vortex, onion and uniform one. In this work we studied possible ground states for a thin spherical shell, which can be considered as a 3D extension of a narrow planar ring.

It has been shown that vortex ground state of magnetic spherical shell consists of two vortices with opposite polarities [3]. Using micromagnetic simulations we studied how the ground state depends on geometrical parameters.

II. MICROMAGNETIC SIMULATION

In order to perform micromagnetic simulations we use MAGPAR code [4] with permalloy material parameters: $A = 1.3 \times 10^{-11} J/m$, $M_s = 8.6 \times 10^5 A/m$, $\alpha = 0.01$. We consider spherical shell (Fig. 1) with inner radius R and thickness h. In order to identify ground state for particular combination R and h we simulate energy minimization procedure starting from vortex and uniform states. After comparison energies for both states we consider the state with the lowest energy as ground state.

The results obtained from simulations demonstrate a continuous transition from vortex to uniform state for spherical shell. Using the spherical frame of reference, we parameterize the normalized magnetization as follows:

$$\boldsymbol{m} = \left(m_r, m_\vartheta, m_\varphi\right) = (cos\Theta, sin\Theta cos\Phi, sin\Theta sin\Phi). \quad (1)$$

We determine angle Φ at the equator ($\vartheta = \pi/2$) as it is shown in Fig. 1. Calculated from simulation values for Φ are displayed in Fig. 2 (red circles) for spherical shell with thickness $h = 10\ nm$ and different radii.

III. SPHERICAL SHELL ENEGRY

In order to an analytical insight we consider a model of a soft ferromagnet, where only exchange and manetostatic interactions are taken into account, and the anisotropy is neglected. Thus, the total energy normalized by $4\pi M_s^2 V$ with $V = \frac{4}{3}\pi R^3$ reads:

$$E = E_{ms} + E_{ex}\ , \quad (2)$$

with E_{ms} being the magnetostatic energy and E_{ex} being the exchange energy.

In order to calculate magnetostatic energy we use general form of magnetostatic functional:

$$E_{ms} = -\frac{3}{8\pi}\int_V d\boldsymbol{r}(\boldsymbol{m}\cdot\boldsymbol{h}^{ms}), \quad (3)$$

where $\boldsymbol{h}^{ms} = \boldsymbol{H}^{ms}/4\pi M_s$ being reduced magnetostatic field.

Exchange energy can be obtained from general exchange functional:

$$E_{ex} = -A\int_V d\boldsymbol{r}(\boldsymbol{m}\cdot\nabla^2\boldsymbol{m}). \quad (4)$$

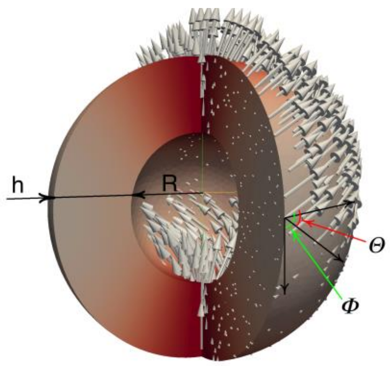

Fig. 1. Schematic of the spherical shell. White arrows correspond to the magnetic distribution.

To describe the continuous transition between vortex and onion states, we use the following Ansatz:

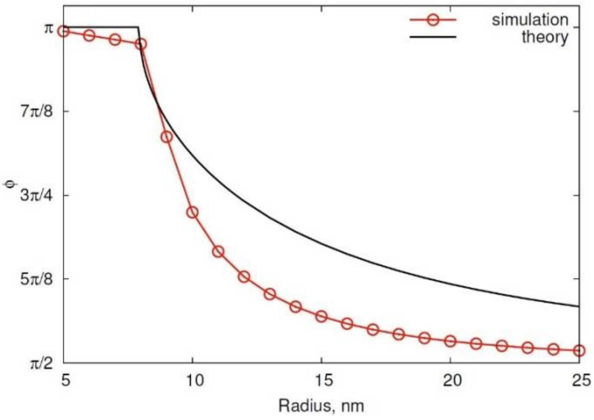

Fig. 2. Angular parameter Φ vs spherical shell radius for the case h = 10 nm. Red circles correspond to the simulation data; black line shows theoretical value for Φ calculated using equation (10).

$$\Theta = \frac{\pi}{2}\left(1 - \frac{\exp(-\vartheta/\vartheta_c) - \exp((\vartheta-\pi)/\vartheta_c)}{1-\exp(-\pi/\vartheta_c)}\right), \Phi = const, \quad (5)$$

where $\vartheta_c = \sqrt{2}\,\lambda_{ex}/R$ determines the vortex core size and $\lambda_{ex} = \sqrt{A/4\pi M_S^2}$ being the exchange length. Ansatz (5) allows us to describe both vortex ($R \gg \lambda_{ex}$) and uniform ($R \le \lambda_{ex}$) states.

Recently the exchange energy for thin curvilinear shell of an arbitrary shape has been calculated in Ref. [5]. Using this result and Ansatz (5) one can obtain an expression for exchange energy in the following form:

$$E_{ex} = G_1^{ex} \cos\Phi + G_0^{ex}, \quad (6)$$

where G_0^{ex} and G_1^{ex} are coefficients which depend only on geometrical parameters, namely radius R and aspect ratio $\varepsilon = h/R$. One can see that exchange energy is linear function of $\cos\Phi$.

Reduced magnetostatic field in (3) can be obtained from equation:

$$\boldsymbol{h}^{ms} = -\nabla\psi(\boldsymbol{r}), \quad (7)$$

where $\psi(\boldsymbol{r})$ is magnetostatic potential. Magnetostatic potential can be calculated from equation:

$$\psi(\boldsymbol{r}) = \int d\boldsymbol{r}' \frac{\lambda(\boldsymbol{r}')}{|\boldsymbol{r}-\boldsymbol{r}'|} + \int dS' \frac{\sigma(\boldsymbol{r}')}{|\boldsymbol{r}-\boldsymbol{r}'|}, \quad (8)$$

where $\lambda = -(\nabla \cdot \boldsymbol{m})/4\pi$ determines volume magnetostatic charge and $\sigma = (\boldsymbol{m} \cdot \boldsymbol{n})/4\pi$ determines surface magnetostatic charges.

Using equation (8) and an expansion of $1/|\boldsymbol{r} - \boldsymbol{r}'|$ over Legendre function, one can calculate the magnetostatic energy for spherical shell in the following form:

$$E_{ms} = G_2^{ms} \cos^2\Phi + G_1^{ms} \cos\Phi + G_0^{ms}, \quad (9)$$

where G_0^{ms}, G_1^{ms} and G_2^{ms} are coefficients which depend only on geometrical parameters R and ε.

Therefore total energy reads:

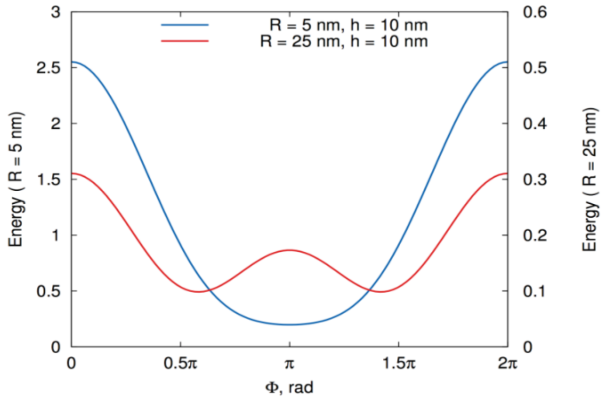

Fig. 3. Energy as a function of angular parameter Φ for spherical shell with thickness 10 nm and different radii.

$$E = G_2 \cos^2\Phi + G_1 \cos\Phi + G_0, \quad (10)$$

with $G_2 = G_2^{ms}$, $G_1 = G_1^{ms} + G_1^{ex}$ and $G_0 = G_0^{ms} + G_0^{ex}$.

One can see that energy functional (10) has two minima for $\Phi = \pi \mp \Phi_0$ (Fig. 3). Since vortex chirality is determined by Φ, two symmetric minimums in Fig. 3 means that two chiralities are equivalent in spherical shell. Value of Φ_0 can be found from equation:

$$\cos\Phi = -G_1/2G_2 \quad (11)$$

Analysis of equation (11) shows that for large radii $\Phi_0 \to \pi/2$, so that vortex is ground state, and for small radii $\Phi_0 \to 0$, what means that vortex state is transformed to uniform one. Critical radius R_c when uniform state becomes ground one ($\Phi = \pi$) can be found from equation:

$$G_1(R_c, \varepsilon) = 2\, G_2(R_c, \varepsilon) \quad (12)$$

Theoretically calculated values for Φ using Eq. (11) are shown in Fig. 2 for different radii. One can see that value for Φ continuously grows while radius decreases, so that vortex distribution transforms into uniform one.

In conclusion, the ground states of spherical shells have been studied both theoretically and by means of micromagnetic simulations. A continuous transformation of the vortex state into onion one with the shell radius decreasing is demonstrated. The critical radius of transition is obtained as a function of the shell thickness.

References

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