Importance of the Internal Shape Mode in Magnetic Vortex Dynamics

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We investigate the motion of a nonplanar vortex in a circular easy-plane magnet with a rotating in-plane magnetic field. Our numerical simulations of the Landau-Lifshitz equations show that the vortex tends to a circular limit trajectory, with an orbit frequency which is lower than the driving field frequency. To describe this we develop a new collective variable theory by introducing additional variables which account for the internal degrees of freedom of the vortex core, strongly coupled to the translational motion. We derive the evolution equations for these collective variables and find limit-cycle solutions whose characteristics are in qualitative agreement with the simulations of the many-spin system.

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Nonlinear excitations play an important role in various physical contexts such as light propagation, charge, and energy transport in condensed-matter physics and biophysics, and Bose-Einstein condensation of dilute atomic gases [1–3]. They also appear in low-dimension magnetic structures like 2D arrays of magnetic nanodisks and dots [4,5], rings [6], and stripes (for reviews, see [5,7,8]). In these systems the topological excitations (domain walls and vortices) determine the static and dynamic properties. For instance, when the size of a magnetic nanodisk is above a critical value, a vortex state is energetically preferable. Its core region exhibits a bell-shaped structure of the spin components perpendicular to the plane of the film [5,7].

Until recently vortex dynamics was studied in the frame of the Thiele collective coordinate approach [9], which considers the vortex as a rigid structure not having internal degrees of freedom (for a review, see Ref. [10]). The idea is to assume a traveling wave solution whose time dependence is only in the coordinates of the vortex. The evolution of these coordinates is then obtained from the field equations using the traveling wave ansatz. However, recent experimental [4,5] and theoretical [11–14] studies indicate phenomena which cannot be understood using this simple approach. Some striking examples are the switching of the vortex polarization by means of an applied in-plane (IP) rotating field [12,13,15] and the cycloidal oscillations of the vortex around its mean path [11,16]. In these examples the dynamics of the vortex center is observed to be strongly coupled with spin waves. One of the first attempts to take into account the internal structure of vortices was presented in Ref. [14], where in order to describe the motion of vortices in an inhomogeneous medium an internal core mode, slaved to the center of vortex motion, was shown to be important.

Here we confirm that the internal degrees of freedom play a crucial role in the dynamics of vortices driven by an external time-dependent magnetic field in a classical spin system. We also present a collective variable (CV) theory describing how the internal mode couples to the translation mode. This theory can be seen as the first generalization to vortices of the CV theory developed for 1D Klein-Gordon kinks by Rice [17,18], which includes as CV the position as well as the width of the kink.

Specifically we study the dynamics of one vortex in the classical 2D Heisenberg easy-plane ferromagnet under the action of an applied spatially homogeneous IP rotating field $\boldsymbol{B}(t) = (B\cos\omega t, B\sin\omega t, 0)$. Numerical simulations show that the vortex center tends to a circular trajectory, which is a limit cycle, stable for a broad range of field parameters (B, ω). The frequency Ω of rotation along the vortex orbit in the limit cycle is much smaller than the frequency ω of the applied field. This limit trajectory cannot be obtained from the Thiele equations for the vortex coordinates $Z(t) \equiv X(t) + iY(t)$. Taking into account only the internal structure of the vortex, in our case the width $l(t)$ of the core, and precession of spins as a whole, in our case the phase $\Psi(t)$, as dynamical variables coupled with the vortex coordinates, $Z(t)$, one can understand the vortex dynamics.

We start with the Heisenberg Hamiltonian for classical spins $\boldsymbol{S}_{\boldsymbol{n}}$ located at sites $\boldsymbol{n}$ of a 2D square lattice, plus a Zeeman interaction with the IP rotating field:

$$\mathcal{H} = -\frac{J}{2}\sum_{(\boldsymbol{n},\boldsymbol{n}')}[\boldsymbol{S}_{\boldsymbol{n}}\cdot\boldsymbol{S}_{\boldsymbol{n}'} - \delta S^z_{\boldsymbol{n}}S^z_{\boldsymbol{n}'}], \tag{1a}$$

$$\mathcal{V}(t) = -\gamma B\sum_{\boldsymbol{n}}[S^x_{\boldsymbol{n}}\cos(\omega t) + S^y_{\boldsymbol{n}}\sin(\omega t)], \tag{1b}$$

where $J > 0$ is the exchange integral, δ is the anisotropy constant $(0 < \delta \ll 1)$, $\gamma = 2\mu_B/\hbar$ is the gyromagnetic

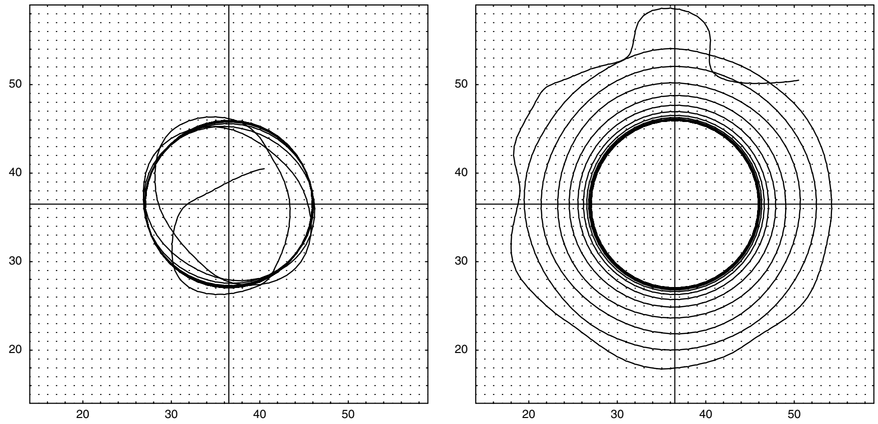

FIG. 1. Two trajectories of a vortex from simulations of the many-spin model (1), in a system of radius $L = 36a$, in the presence of a rotating field ($\nu = 0.02$, $b = 0.004$), damping $\epsilon = 0.01$, and anisotropy $\delta = 0.08$. For this field, all trajectories converge to the same circle independently of the vortex initial position, providing it is not too close to the system border.

ratio, and the sum is over all lattice sites $\boldsymbol{n}$ and its nearest neighbors $\boldsymbol{n}'$. A magnetostatic (dipolar) interaction term was taken into account by its effective action on the anisotropy and conditions at the borders. For very thin and extended magnetic films this term leads to a renormalization of the anisotropy constant and to free boundary conditions [19].

The continuum approach for the model (1) is formulated in terms of the canonically conjugated variables $m(\boldsymbol{x}, t) \equiv m^z(\boldsymbol{x}, t) = S^z(\boldsymbol{x}, t)/S$ and $\phi(\boldsymbol{x}, t)$, with $m^x + im^y = \sqrt{1 - m^2}\exp(i\phi)$. The total energy reads

$$\mathcal{E} = \frac{JS^2}{2}\int d^2x\left[\frac{(\boldsymbol{\nabla} m)^2}{1 - m^2} + (1 - m^2)(\boldsymbol{\nabla}\phi)^2 + \frac{m^2}{l_0^2}\right] - \frac{\gamma BS}{a^2}\int d^2x\sqrt{1 - m^2}\cos(\phi - \omega t). \qquad (2)$$

where a is the lattice constant and $l_0 = a/\sqrt{4\delta} \gg a$ is the characteristic magnetic length of the model. The dynamics of the system (2) is governed by the Landau-Lifshitz equations including Gilbert damping,

$$\frac{\partial\phi}{\partial t} = \frac{a^2}{S}\frac{\delta\mathcal{E}}{\delta m} + \frac{\epsilon}{(1 - m^2)}\frac{\partial m}{\partial t}, \qquad (3a)$$

$$\frac{\partial m}{\partial t} = -\frac{a^2}{S}\frac{\delta\mathcal{E}}{\delta\phi} - \epsilon(1 - m^2)\frac{\partial\phi}{\partial t}, \qquad (3b)$$

with a damping constant ϵ. These equations can be derived from the Lagrangian

$$\mathcal{L} = -\frac{S}{a^2}\int d^2x(1 - m)\frac{\partial\phi}{\partial t} - \mathcal{E} \qquad (4)$$

plus a dissipative function

$$\mathcal{F} = \frac{\epsilon S}{2a^2}\int d^2x\left[\frac{1}{1 - m^2}\left(\frac{\partial m}{\partial t}\right)^2 + (1 - m^2)\left(\frac{\partial\phi}{\partial t}\right)^2\right]. \qquad (5)$$

The simplest nonlinear excitation in the system is a vortex state with an out-of-plane core [20,21]. For a circular system of radius L and free (Neumann) boundary conditions, a static vortex solution is approximately given by [16]

$$m(z) = m_0(|z - Z|/l_0), \qquad (6a)$$

$$\phi(z) = \arg(z - Z) - \arg(z - \bar{Z}) + \arg Z, \qquad (6b)$$

where $z = x + iy$ is a point in the XY plane and $Z = R\exp(i\Phi)$ is the coordinate of the vortex center. The "image" vortex at $\bar{Z} = ZL^2/R^2$ is added to satisfy the boundary conditions, and the last term in (6b) is a constant inserted to have a correct limit for $L \to \infty$. The function $m_0(\rho)$, with $\rho \equiv |z|/l_0$, is the solution of a boundary problem which can be obtained numerically [22], with $m_0(0) = \pm 1$, $m_0(\rho \to \infty) \to 0$.

To investigate the vortex dynamics in the presence of the rotating field, we integrated numerically the discrete Landau-Lifshitz-Gilbert equations for the model (1),

$$\frac{d\boldsymbol{S_n}}{dt} = -\boldsymbol{S_n} \times \frac{\partial(\mathcal{H} + \mathcal{V})}{\partial\boldsymbol{S_n}} - \frac{\epsilon}{S}\boldsymbol{S_n} \times \frac{d\boldsymbol{S_n}}{dt}. \qquad (7)$$

The details of the simulations are given in Ref. [12]. To avoid switching phenomena observed in Ref. [12] we restricted ourselves to the case when $m_0(0) > 0$ and $\omega > 0$. We have fixed the anisotropy $\delta = 0.08$ ($l_0 \approx 1.77a$) and the damping $\epsilon = 0.01$ and varied the parameters (b, ν, L) where $b = \gamma B/JS$ and $\nu = \omega/JS$.

Given a combination of the parameters (ν, b) of the field, the radius L of the system and the damping ϵ, we have observed that either the vortex escapes from the system through the border or it stays inside for all times. In the latter case, it can approach a limit cycle for a broad range of the field parameters (for $L = 36a$, e.g., $0.002 < b < 0.008$ and $0.02 < \nu < 0.07$). Figure 1 shows two vortex trajectories starting from different positions and converging to the same circle. To exist, the limit cycle needs both magnetic field and damping: once it is attained, switching off or changing either of them destroys immediately the circular trajectory. If the amplitude b is not large enough, the damping will dominate and the vortex will escape from the system following a spiral. If the amplitude is too large (for given ν and L) the vortex will also escape due to an effective drift force exerted by the field. This is also the case when the frequency is very small, such that the field is practically static. If both b and ν are too large, the field can destroy the excitation creating many-spin waves and new vortices can be generated at the boundary. All these extreme cases constrain the size and shape of the regime of parameters (b, ν) where the circular limit cycles appear, for a given L (more details will be given in a forthcoming paper). The limit cycles exhibit two main features: (i) the orbit frequency Ω is smaller than the driving frequency ω, and (ii) the orbit radius R increases linearly with the size L of the system.

To describe analytically the observed vortex dynamics, a standard procedure due to Thiele [9] is to use the traveling wave approach, assuming $\boldsymbol{m} = \boldsymbol{m}[z - Z(t)]$. At zero magnetic field, this method describes very well the dynamics of the vortex: without dissipation it undergoes a circular motion [10] while damping makes it exit the system following a spiral trajectory. This simple approach fails with a rotating magnetic field because the field excites low-frequency quasi Goldstone modes [12,15], which can couple with the translation mode [13]. Therefore we generalize the collective variable approach by the following Ansatz:

$$m(z,t) = m_0\left(\frac{|z - Z(t)|}{l(t)}\right), \tag{8a}$$

$$\phi(z,t) = \arg[z - Z(t)] - \arg[z - \bar{Z}(t)] + \arg Z(t) + \Psi(t), \tag{8b}$$

which describes a mobile vortex structure like (6), including a precession of the spins as a whole [through $\Psi(t)$] and dynamics of the vortex core [through $l(t)$].

We found it convenient to use, instead of $l(t)$, the z component of the total spin,

$$M(t) = \frac{S}{a^2}\int d^2x m(z,t) = M_0\left[\frac{l(t)}{l_0}\right]^2, \tag{9}$$

which is related to the total number of "spin deviations" or "magnons" bound in the vortex [23]. Here $M_0 = Sn_0(l_0/a)^2$ is related to the characteristic number of magnons bound in the *static* vortex, and $n_0 = 2\pi \times \int_0^\infty \rho d\rho m_0(\rho) \approx 8.63$. Note that without dissipation and for zero field, M is a constant of motion.

To construct the equations of motion for the collective variables $X_i = \{R(t), \Phi(t), M(t), \Psi(t)\}$ we insert the Ansatz (8) in the "microscopic" Lagrangian (4) and dissipation function (5) and calculate the integrals to get an effective Lagrangian

$$\mathcal{L} = M\dot{\Psi} - \frac{\pi S}{a^2}R^2\dot{\Phi} - \pi JS^2 \ln\frac{L^2 - R^2}{l_0 L} + \frac{\pi JS^2}{2}\ln\frac{M}{M_0} - \frac{\pi JS^2}{2}\frac{M}{M_0} - \frac{\pi LS\gamma B}{a^2}R\cos(\Phi + \Psi - \omega t), \tag{10}$$

and an effective dissipation function

$$\mathcal{F} = \frac{\epsilon\pi S}{2a^2}\left[(\dot{R}^2 + R^2\dot{\Phi}^2)\ln\frac{L}{l_0} + L^2\dot{\Psi}^2 + 2R^2\dot{\Phi}\dot{\Psi} + \frac{C\dot{M}^2}{M}\right] \tag{11}$$

with

$$C \equiv \frac{1}{2}\frac{l_0^2}{M_0}\int_0^\infty\left(\frac{m_0'(\rho)}{1 - m_0(\rho)^2}\right)^2\rho^3 d\rho \approx 0.48\frac{l_0^2}{M_0}.$$

In Eqs. (10) and (11) we assumed that the vortex never comes close to the boundary ($R \ll L$). The first two terms in Eq. (10) result from the first integral ("gyrotropic" term) in the Lagrangian (4), the next three terms result from the first integral in the energy (2), and the last term in Eq. (10) comes from the Zeeman interaction in (2). The equations of motion are then obtained from the Lagrange equations

$$\frac{\partial\mathcal{L}}{\partial X_i} - \frac{d}{dt}\left(\frac{\partial\mathcal{L}}{\partial\dot{X}_i}\right) = \frac{\partial\mathcal{F}}{\partial\dot{X}_i}.$$

After scaling the time to $\tau = JSt$ and defining as before $b = \gamma B/(JS)$ and $\nu = \omega/(JS)$, we obtain the final collective variable equations

$$R_\tau = \frac{\epsilon R}{2}\left(\Phi_\tau \ln\frac{L}{l_0} + \Psi_\tau\right) - \frac{bL}{2}\sin(\Phi + \Psi - \nu\tau), \tag{12a}$$

$$\Phi_\tau = \frac{a^2}{L^2 - R^2} - \frac{\epsilon R_\tau}{2R}\ln\frac{L}{l_0} - \frac{bL}{2R}\cos(\Phi + \Psi - \nu\tau), \tag{12b}$$

$$\frac{1}{\pi S}M_\tau = -\frac{\epsilon}{a^2}(L^2\Psi_\tau + R^2\Phi_\tau) + \frac{bLR}{a^2}\sin(\Phi + \Psi - \nu\tau), \tag{12c}$$

$$\frac{1}{\pi S}\Psi_\tau = \frac{1}{2}\left(\frac{1}{M_0} - \frac{1}{M}\right) + \frac{\epsilon C}{a^2}\frac{M_\tau}{M}, \tag{12d}$$

where the subindex $(\cdots)_\tau$ denotes the derivative with respect to the scaled time.

The set of Eqs. (12) describes the main features of the observed vortex dynamics, and yields the circular limit cycle for the trajectory of the vortex center; see Fig. 2. The radius and frequency of this limit cycle are of the same order as those of our simulation results. In particular, in the limit cycle, $\Phi = \Omega t$, and Ω is smaller than the driving frequency ω. Equations (12a) and (12b) reduce to the Thiele equations for the coordinates (R, Φ) of the vortex center when M and Ψ are omitted, and in this case no stable closed orbit is possible. Only including the internal degrees of freedom (M, Ψ) one can obtain a stable limit cycle. Its characteristics can be found by setting

$$R_\tau = M_\tau = 0, \qquad \Phi_\tau + \Psi_\tau = \nu.$$

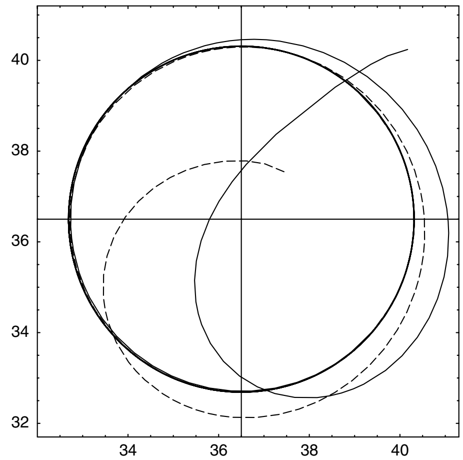

FIG. 2. Two trajectories of a vortex from the collective variable Eqs. (10)–(12), starting from different initial positions with field parameters $\nu = 0.02$, $b = 0.004$. Dashed line: $R(0) = \sqrt{2}$, $\Phi(0) = \pi/4$, $\Psi(0) = 3\pi/2$. Solid line: $R(0) = 5.3$, $\Phi(0) = \pi/4$, $\Psi(0) = \pi/2$. Other parameters: $\epsilon = 0.01$ and $\delta = 0.08$. System radius: $L = 36a$.

This yields an asymptotic behavior of the orbit radius, $R \sim bL/\Omega$, which is valid for large system sizes, $L \gg l_0$. This linear dependence of R on L is also observed in our simulations.

In conclusion, we developed a new collective variables approach which describes the vortex dynamics under a periodic driving, taking into account internal degrees of freedom. To our knowledge, it is the first time that an interplay between internal and external degrees of freedom, giving rise to the existence of stable trajectories, is observed in the case of 2D magnetic structures. This collective variable approach is very general and could describe the dynamics of different 2D nonlinear excitations, e.g., topological solitons in 2D easy-axis magnets [24].

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