Domain Wall Dynamics at the Local Wire Bend

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I. Introduction

The interaction of curvature and topologically nontrivial magnetization structures (e.g. domain walls) attract growing interest in fundamental studies and applications [1-4]. Domain walls are topologically stable structures which can be used as key elements of nonvolatile magnetic memory devices [1-3]. In the racetrack domain wall magnetic memory [2] the sequence of bits of information in a nanowire is coded by sequence of magnetic domains separated by the domain walls of head-to-head and tail-to-tail types [5]. However, curvature parameters are an important characteristic of the wire in fabrication of memory devices. Vertical configuration of the racetrack magnetic memory consists of U-shaped nanowires [2] and therefore the domain wall properties on the curvilinear segment of the wire are crucial.

In current study we apply the theory, developed in Ref. 6, to describe the curvature induced pinning of the transversal domain wall. The linear eigenmodes winthin the pinning potential are studied by means of the collective variable approach.

II. Domain Wall Dynamics

We consider a plane curve wire with round cross-section. Such wire can be parameterized in the following way:

$$\boldsymbol{r}(s,\chi,\rho) = \boldsymbol{\gamma}(s) + \rho\cos\chi\, \boldsymbol{e}_{\rm n}(s) + \rho\sin\chi\, \boldsymbol{e}_{\rm b}(s), \qquad (1)$$

where $\boldsymbol{r}$ defines space domain around the curved wire $\boldsymbol{\gamma}(s)$ with s being the arc length, $0 \le \rho \le R$ and $0 \le \chi \le 2\pi$ being cross-section wires polar coordinate. We use Frennet-Serret basis ($\boldsymbol{e}_{\rm t}$, $\boldsymbol{e}_{\rm n}$, $\boldsymbol{e}_{\rm b}$) with $\boldsymbol{e}_{\rm t} = \boldsymbol{\gamma}'(s)$, $\boldsymbol{e}_{\rm n} = \boldsymbol{\gamma}''(s)/\kappa(s)$, and $\boldsymbol{e}_{\rm b} = [\boldsymbol{e}_{\rm t}, \boldsymbol{e}_{\rm n}]$ being tangential, normal, and binormal unit vectors respectively. Here the wire curvature $\kappa(s) = |\boldsymbol{\gamma}''(s)|$. Here and below prime denotes to the derivatives with respect to s.

Using the Frennet-Serret basis one can introduce the angular magnetization parameterization

$$\boldsymbol{m} = \cos\theta\, \boldsymbol{e}_{\rm t} + \sin\theta\cos\varphi\, \boldsymbol{e}_{\rm n} + \sin\theta\sin\varphi\, \boldsymbol{e}_{\rm b}, \qquad (2)$$

where $\boldsymbol{m} = \boldsymbol{M}/{\rm M}_{\rm s}$ is normalized magnetization unit vector with ${\rm M}_{\rm s}$ being the saturation magnetization.

Magnetization dynamics can be described by the phenomenological Landau-Lifshitz equations

$$-\sin\theta\, \partial\theta/\partial t = \omega_0\, \delta E/\delta\varphi, \qquad \sin\theta\, \partial\varphi/\partial t = \omega_0\, \delta E/\delta\theta, \qquad (3)$$

where frequency $\omega_0 = 4\pi\gamma_0 Ms$ determines the characteristic time scale with γ_0 being the gyromagnetic ratio, E is a total energy normalized by the $4\pi Ms^2$.

Let us describe the magnetic properties of the wire. We base our analysis on the following assumptions:

(i) we suppose the magnetization spatial one-dimensionality, $\boldsymbol{m} = \boldsymbol{m}(s,t)$ (the case when thickness $2R$ does not exceed the characteristic magnetic length);

(ii) we consider a magnet with the dipolar interaction reduced to the effective easy-tangential anisotropy [7,8].

The normalized energy of the system can be written as:

$$E = S\int[l^2\, \varepsilon_{\rm ex} - k_{\rm t}\cos^2\theta]{\rm d}s, \qquad (4)$$

here S is cross-section area, $l=\sqrt{(A/4\pi Ms^2)}$ is exchange length with A being exchange constant, $\varepsilon_{\rm ex}$ is the exchange energy density, and $k_{\rm t}=K/4\pi Ms^2>0$ is the dimensionless anisotropy constant.

In terms of the angular parameterization (2) the exchange energy density of plane wire has form [6]

$$\varepsilon_{\rm ex} = (\theta' + \kappa\cos\varphi)^2 + (\varphi'\sin\theta - \kappa\cos\theta\sin\varphi)^2. \qquad (5)$$

Let us analyze the static case of Eq. (3)

$$\delta E/\delta\varphi = 0 \qquad \delta E/\delta\theta = 0. \qquad (6)$$

In this case the solution function θ can be obtained as a solution of the equation

$$\theta''\, l^2\, k_{\rm t}^{-1} - \sin\theta\cos\theta = l^2\, k_{\rm t}^{-1}\, \kappa'\cos\varphi_0 \qquad (7)$$

and $\varphi = \varphi_0 = 0, \pi$.

For the case $\kappa'\equiv 0$ (e.g. straight wire) Eq. (7) has the well-known domain wall solution $\theta = 2\arctan[\exp[p(s-q)/\Delta_0]]$ of head-to-head ($p = 1$) or tail-to-tail($p = -1$) types. Here q determines domain wall position and $\Delta_0 = l\, k_{\rm t}^{-1/2}$ is domain wall width. In the following, we consider a case of a localized curvature, i.e. $\kappa(\pm\infty) = 0$ and $\kappa'(\pm\infty) = 0$, and we assume that the curvature has weak influence on domain wall form. Therefore to study dynamical properties of the transverse domain wall we use collective variable approach [9,10] based on the simple q-Φ model [11]

$$\theta = 2\arctan[\exp[p(s-q(t))/\Delta_0]], \qquad \varphi = \Phi(t). \qquad (8)$$

The domain wall position q and phase Φ, which determines orientation of the transversal magnetization components, are a canonically conjugated pair of collective variables.

The equations of motion (3) are the Euler-Lagrange equations for the Lagrange function

$$L = -S\int\varphi\sin\theta\, \partial\theta/\partial t\, {\rm d}s - E \qquad (9)$$

In terms of the collective variables, the effective equations of motion read

$$\omega_0^{-1}\mathrm{d}q/\mathrm{d}t = l^2\,\Delta_0^{-1}\sin\Phi\,(f_0 + q^2 f_1(2\Delta_0^2)^{-1}) + l^2\,\Delta_0^{-1}\,g_0\sin\Phi\cos\Phi,$$

$$\omega_0^{-1}\mathrm{d}\Phi/\mathrm{d}t = f_1 l^2 q\Delta_0^{-3}\cos\Phi. \qquad (10)$$

In current study we want to analyze the linear dynamics in the vicinity of the equilibrium position. With this purpose we introduce small deviations in the way $q(t) = q_0 + \tilde{q}(t)$ and $\Phi(t) = \Phi_0 + \phi(t)$. In this case linearized equations of motion (10) reads

$$\mathrm{d}\tilde{q}/\mathrm{d}t = \omega_0\, l^2\,\Delta_0(f_0-g_0)\phi, \qquad \mathrm{d}\phi/\mathrm{d}t = -\,\omega_0\, l^2\,\Delta_0^{-3} f_1\tilde{q}. \qquad (11)$$

Eqs. (11) have a solution in harmonic oscillations $\tilde{q}=\tilde{q}_a\sin(\Omega t+\delta_0)$, $\phi=\phi_a\cos(\Omega t+\delta_0)$, with frequency

$$\Omega = \omega_0\, l^2\,\Delta_0^{-2}((f_0-g_0)f_1)^{1/2}, \quad f_1=\Delta_0^2\int\kappa''/\cosh(s/\Delta_0)\mathrm{d}s,$$

$$f_0=\int\kappa/\cosh(s/\Delta_0)\mathrm{d}s, \qquad g_0=\Delta_0\int\kappa^2/\cosh^2(s/\Delta_0)\mathrm{d}s. \qquad (12)$$

For the limit case $\kappa\Delta_0\to 0$ the value of eigenfreauency reads

$$\Omega = \omega_0\, l^2\,\pi(|\kappa''(q_0)\kappa(q_0)|)^{1/2}. \qquad (13)$$

To check the obtained results we consider a parabolic wire, with central line

$$\gamma(s) = (x, \kappa_0 x^2/2, 0), \qquad (14)$$

where κ_0 is maximal curvature value at the wire bend. The curvature of this wire defined in the following way $\kappa=\kappa_0(1+\kappa_0^2 x^2)^{-3/2}$.

In accordance to (14) the linear oscillations of domain wall in parabolic wire is

$$\Omega = \omega_0\, l^2\,\Delta_0^{-2}(|F_1(\kappa_0\Delta_0)\,F_2(\kappa_0\Delta_0)|)^{1/2},$$

$$F_1(x)=3x^2\int(5\xi^2-1)(1+\xi^2)^{-4}\mathrm{sech}(f(\xi)/x)\mathrm{d}\xi,$$

$$F_2(x)=\int[1-x(1+\xi^2)^{-3/2}\mathrm{sech}(f(\xi)/x)](1+\xi^2)^{-1}\mathrm{sech}(f(\xi)/x)\mathrm{d}\xi,$$

$$f(\xi)=2^{-1}[\xi(1+\xi^2)^{1/2}+\sinh^{-1}(\xi)]. \qquad (15)$$

For the case of a narrow domain wall ($\Delta_0\kappa_0\to 0$) one can use the asymptotics (13), which results in

$$\Omega \approx \omega_0\pi(\kappa_0 l)^2\sqrt{3}. \qquad (16)$$

III. Simulations

In order to verify our analytical results we performed two types of simulation: (*A*) simulation of the magnetization dynamics of a parabolic-shaped wire with finite thickness; (*B*) simulation of the magnetization dynamics of a parabolic-shaped chain of discrete magnetic moments.

A. Wire with finite thickness

The simulation of magnetization dynamics in the parabolic wire with finite thickness, parameterized by Eq. (1), was performed by using the NMAG code [11] with the Permalloy parameters: exchange constant A = 1.3·10^{-11} J/m, saturation magnetization M_s = 8.6·10^5 A/m, and damping coefficient α = 0.01. Thermal effects and anisotropy are neglected. We start with the parabolic wire with radius R = 5 nm and length 1000 nm, with the curvature κ_0 is varied in the range from 0.005 nm^{-1} up to 0.05 nm^{-1}. The volume domain of the sample is discretized using irregular tetrahedral mesh with cell size about 1.75 nm.

The observation of dynamical properties of domain wall on parabolic wire with finite thickness was done in two steps. Firstly, we relaxed our domain wall in the parabolic wire in the spatially uniform magnetic field $\boldsymbol{b}$ = (0, 0, 25) mT in overdamped regime (α = 0.1). In the second step we switch off the magnetic field and simulate the magnetization dynamics with the damping parameter close to natural (α = 0.01). Then the time Fourier transform is performed for the time-dependent domain wall position $q(t)$. The frequency which corresponds to the maximum of the Fourier signal is marked by a symbol for a given κ_0, see the Fig. 1.

B. Wire formed with the chain of discrete magnetic moments

Also we studied the dynamical properties of discrete chain of magnetic moments $\boldsymbol{m}_i$, with $i = 1\ldots N$, aligned along the parabolic line. The moments are aligned equidistantly with the step size Δs. Like for the wire with finite thickness only three magnetic interactions are taken into account, namely exchange (with $l=3\Delta s$), dipole-dipole and Zeeman contributions. The total number of the moments is fixed $N=76$ while the curvature parameter κ_0 is varied but the restriction is fixed $\kappa_0\Delta s<<1$. The dynamics of this system is described as a set of $3N$ Landau-Lifshitz equations witch solved numerically, in details this procedure is described in the Ref.12.

The domain wall dynamics are studied in the same way as for the wires described above. The resulting eigenfrequencies are shown in Fig. 1.

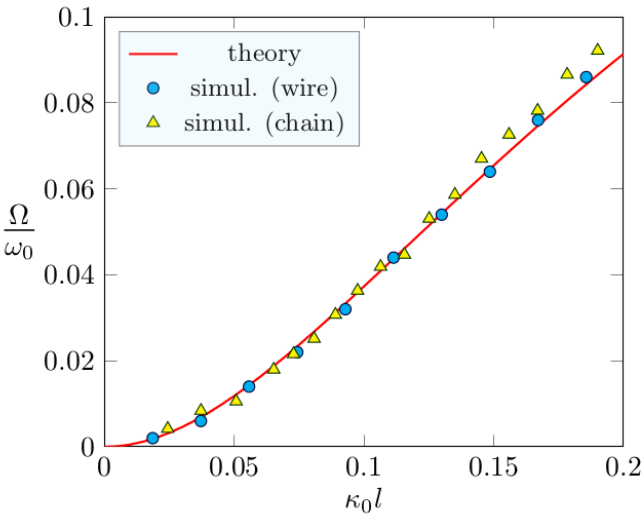

Fig. 1. Eigenfrequency of domain wall oscilations in the bend of parabolic wire. Solid line correspond to the Eq. (15). Markers shows the results of numerical simulations for nanowires (dots) and discrete chains of magnetic moments (triangles).

IV. Conclusion

In conclusion, we have presented a general approach of the domain wall dynamics at a wire bend with localized curvature. We have analytically found dependence of domain wall eigen frequency on the curvature. For a certain case of parabolic wire bend we performed full-scale micromagnetic simulations to check our analytical predictions. The results of simulations are in good agreement and demonstrate that the approximation of magnetostatic interaction by the effective easy-tangential anisotropy is physically sound for a domain wall dynamics in the thin nanowires.

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