Comment on "Magnon wave forms in the presence of a soliton in two-dimensional antiferromagnets with a staggered field"

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Very recently, Fonseca and Pires [Phys. Rev. B **73**, 012403 (2006)] have studied the soliton-magnon scattering for the isotropic antiferromagnet and calculated "exact" phase shifts, which were compared with the ones obtained by the Born approximation. In this Comment we correct both the soliton and magnon solutions and point out the way how to study correctly the scattering problem.

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The soliton-magnon interaction in two-dimensional (2D) magnets is a subject of intensive studying for more than 10 years. In a recent paper Fonseca and Pires[1] considered the soliton-magnon scattering problem in an isotropic antiferromagnet in the presence of a staggered magnetic field.

Dynamics of the antiferromagnet in the presence of the staggered field can be described on the basis of a simple generalization of the σ-model for the sublattice magnetization unit vector $\boldsymbol{n}$.[1] Using the angular parametrization, $\boldsymbol{n}=\{\sin\theta\cos\phi,\sin\theta\sin\phi,\cos\theta\}$, the energy of the system takes the form[7]

$$E=\frac{J}{2}\int d^2x\left\{\frac{1}{c^2}\left(\frac{\partial\theta}{\partial t}\right)^2+(\boldsymbol{\nabla}\theta)^2+\sin^2\theta\left[\frac{1}{c^2}\left(\frac{\partial\phi}{\partial t}\right)^2+(\boldsymbol{\nabla}\phi)^2\right]-2h\cos\theta\right\}. \tag{1}$$

In the absence of the field h, one has a standard σ-model with the well-known soliton solution,[2] the so-called Belavin-Polyakov (BP) soliton. The scattering of magnons by the BP soliton in different isotropic magnets, in particular, antiferromagnets was solved in Ref. 3.

The presence of the field drastically changes the picture: the simplest BP soliton does not exist. First of all, one needs to look for the soliton solutions in the system with a staggered field h. For determinacy we consider the case $h>0$, which provides the ground state $\theta_0=0$.

To describe the soliton structure Fonseca and Pires[1] proposed the following distribution of $\boldsymbol{n}_s$:

$$\theta_s=\theta_s(r),\quad \phi_s=q\varphi-\Omega t, \tag{2a}$$

where r and φ are the polar coordinates in the XY plane, $k_0=\Omega/c$ and $q\in\mathbb{Z}$. The function θ is the solution of the differential problem

$$\frac{d^2\theta_s}{dr^2}+\frac{1}{r}\frac{d\theta_s}{dr}+\left(k_0^2-\frac{q^2}{r^2}\right)\sin\theta_s\cos\theta_s=h\sin\theta_s, \tag{2b}$$

$$\theta_s(0)=\theta_s(\infty)=0. \tag{2c}$$

Note that according to Eq. (2c) the possible distribution of $\boldsymbol{n}_s$ has no π_2-topological properties; the topological charge

$$Q=\frac{\epsilon_{ij}}{8\pi}\int d^2x\boldsymbol{n}\cdot[\partial_i\boldsymbol{n}\times\partial_j\boldsymbol{n}]=\frac{q}{2}[\cos\theta(0)-\cos\theta(\infty)]$$

takes only the trivial value. Thus we will not limit ourself by the boundary condition (2c) and consider general properties of Eq. (2b). One of the fixed points of Eq. (2b) with $\theta=\arccos H$ corresponds to the nonlocal vortexlike solution. Such vortex solution was considered by Fonseca and Pires,[1] see Eq. (9). However, the energy of such solution does not have a logarithmic divergence like Eqs. (12) and (13) from Ref. 1: the correct form is mainly determined by the term

$$\frac{J}{2c^2}\int d^2x\sin^2\theta\left(\frac{\partial\phi}{\partial t}\right)^2\propto R^2$$

and diverges as the system area R^2, so the precessional vortex solution is not preferable.

Excluding the vortex solution, we are interested in localized solutions, which satisfy boundary conditions[4] $\theta(0)=\pi p$, $\theta(\infty)=0$ with $p\in\mathbb{Z}$. Analysis shows that solutions of such type decay far from the origin not fast enough to be square integrable, $\theta\propto\cos(\kappa r)/\sqrt{\kappa r}$, where $\kappa=\sqrt{k_0^2-h}$. Hence the energy of such solutions also diverges, $E\propto R$.

Therefore one can conclude that the differential problem (2b) has *only* the trivial solution $\theta_s(r)=0$; it makes no sense to consider some nontrivial distribution in the ϕ-field because the soliton does not exist.

Let us mention that the soliton structure can be easily found for the case of the uniform field. Both for the ferro- and antiferromagnet, the soliton can be described by the Belavin-Polyakov solution with an additional precession

$$\tan\frac{\theta_s}{2}=\left(\frac{R}{r}\right)^{|q|},\quad \phi_s=q\chi-g\mu_B Ht. \tag{3}$$

The scattering problem for such a soliton takes exactly the same form as without field.[3] Note also that the scattering problem for vortices in easy-plane magnets under the actions of uniform field was studied in detail by Ivanov and Wysin.[5] However, this problem is quite different and much more complicated than for isotropic magnets.

Let us consider now the scattering problem for the system in a staggered field. To study magnons on the soliton background, Fonseca and Pires[1] neglect the out-of-plane soliton structure, $\theta_s=0$, which corresponds to our conclusion about the absence of the soliton solution. In order to consider magnons in a presence of the soliton, the following ansatz is involved: $\theta(\boldsymbol{r},t)=\theta_s(\boldsymbol{r})+\eta(\boldsymbol{r},t)$, where the trivial background

$$\theta_s(r)=0, \quad \phi_s(\varphi,t)=q\varphi-\Omega t \tag{4}$$

is chosen in accordance to Ref. 1.

The reasonable question is how the "soliton" (4), which is simply a ground state, can scatter magnons? Authors chose the plane-wave solution of the form $\eta(\boldsymbol{r},t)=\exp(i\boldsymbol{k}\cdot\boldsymbol{r}-i\omega t)$, which is not a correct mathematical object, because the real scalar η cannot be identified with the complex exponent. The correct form is the real quantity $\eta(\boldsymbol{r},t)=A\cos(\boldsymbol{k}\cdot\boldsymbol{r}-\omega t)$, ϕ =const, which describes the linearly polarized spin wave. However, this linearly polarized wave is not compatible with the solution (4): after the substitution into Eq. (5) of Ref. 1, one can obtain the following equation:

$$\frac{q}{r^2}\frac{\partial\eta}{\partial\varphi}=-\frac{\Omega}{c^2}\frac{\partial\eta}{\partial t},$$

which cannot be solved together with Eq. (14) from Ref. 1, but authors do not take it into account. This causes also the wrong dispersion law (15a).

The correct way is to consider the circular polarized spin wave of the form $\theta=\text{const}\ll 1$, $\phi=\boldsymbol{k}\cdot\boldsymbol{r}-\omega t$, which has the same symmetry as a "soliton" solution (4). After that the magnons on the soliton background are described by the linear corrections both to θ and ϕ components and the magnon solution on the background (4) has the form similar to Eq. (17) from Ref. 1 for both corrections. However, instead of nonanalytic dependence $\mu=\sqrt{n^2+q^2}$, the correct index of the Bessel function has a form $\mu=|n+q|$, see, e.g., Ref. 3. In the case of the solution (4) the role of the q-term is the redefinition of the azimuthal quantum numbers n. Therefore the "exact solution" [Eq. (17) of Ref. 1] of the scattering problem should be reexamined. We want to stress also that the Born approximation is *not* adequate for the soliton-magnon scattering problem, see a discussion in Ref. 6.

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[7] We reproduce here the model (3) from Ref. 1, fixing several typos.