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Soliton–Magnon Scattering and Spin Wave Modes for a Small Magnetic Particle in the Vortex State

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Abstract
The spectrum of eigenmodes of a small disk–shaped ferromagnetic particle in the vortex state is studied for weakly anisotropic easy–plane ferromagnet. An exact analytical solution is constructed in the most interesting case of the modes with the lowest frequencies; such eigenfrequencies can be anomalously low in comparison with particles in homogeneous magnetization. For the rest modes the qualitative analysis by the vortex–scattering problem and numerical methods is used; the splitting of doublets with opposite azimuthal quantum numbers *m* is predicted.

1. Introduction
Magnetic ordering manifests itself very brightly in magnetic resonances. Eigenmodes of magnetization of ellipsoid–shaped small particle of a ferromagnet (FM) were studied by Kittel more than 50 years ago [1]; this theory explains very well frequencies of FM resonance.

Now resonance measurements are widely used to study novel magnetic materials containing such particles, e.g. granular magnetic materials and ferroliquids [2]. In such research, the distribution of magnetisation in one particle was taken as homogeneous. However, it is well known that a small particle of FM can contain spin inhomogeneities; in particular, so–called vortex state can be realized [3].

In this paper we consider thin disk–shaped FM particle in the vortex state. We study weakly anisotropic easy–plane FM in two limit cases: when the particle size R is greater or smaller than characteristical scale (“magnetic length”) Δ_0. In such cases the vortex state can be simulated by the Belavin–Polyakov soliton or out–of–plane vortex solutions respectively. We analyse the eigen-spectrum of normal modes: an exact analytical solution was constructed in the most interesting case of the mode with the lowest frequency; for the rest ones we used the qualitative analysis by the vortex–scattering problem and numerical methods.

2. The Model
The macroscopic dynamics of the classical FM follows the Landau–Lifshitz equations for the normalized magnetization $\vec{m}$. In angular variables, $m_x + i m_y = \sin\theta \exp(i\phi)$, $m_z = \cos\theta$, these equations have the form

$$\nabla^2\theta + \sin\theta\cos\theta\left(\frac{1}{\Delta_0^2} - (\nabla\phi)^2\right) = \frac{\sin\theta}{D}\cdot\frac{\partial\phi}{\partial t}, \quad \nabla\left(\sin^2\theta\nabla\phi\right) = -\frac{\sin\theta}{D}\cdot\frac{\partial\theta}{\partial t},$$

where $\Delta_0 = \sqrt{A/K} >> a$, A the exchange constant, K the energy of the easy–plane anisotropy, a the lattice constant, and D the spin–wave stiffness.

For the homogeneous ground state (magnetisation vectors are parallel and confined to the easy–plane) the 2D model has well-known magnon excitations with the gapless dispersion law, linear for finite *K* and quadratic at $K \to 0$,

$$\omega(\vec{k}) = \sqrt{c^2k^2 + \left(Dk^2\right)^2},$$

where $k = \left|\vec{k}\right|, \vec{k}$ is the magnon wave vector, and $c = D/\Delta_0$ is the phase speed of magnons.

Let us consider disk–shaped particle of size *R*. It was found by Usov and Peschany [3] that such a particle can be in the vortex state, where the magnetisation in the boundary is directed along the boundary, i.e. $\theta(R) = \pi/2$, $\phi = \pi/2 + \chi$. This is the same distribution as for 2d solitons with unit topological charge [4], described by the formulae

$$\theta = \theta_0(r), \quad \phi = \phi_0 \equiv \pi/2 + \chi,$$

where r and χ are polar coordinates. The function $\theta_0(r)$ is the solution of a nonlinear ordinary differential equation

$$\frac{d^2\theta_0}{dr^2} + \frac{1}{r}\cdot\frac{d\theta_0}{dr} + \left(\frac{1}{\Delta_0^2} - \frac{1}{r^2}\right)\sin\theta_0\cos\theta_0 = 0 \tag{1}$$

with boundary conditions $\theta_0(0) = 0, \theta_0(R) = \pi/2$, which can be analysed in two limiting cases.

When the system size $R >> \Delta_0$, there is an out–of–plane vortex (OPV) in the magnet with and exponential decaying of m_z far from the vortex core Δ_0. In this case Eq. (1) can be solved numerically only, but the approximate solution $\cos\theta_0(r) \approx \sin\left(\frac{\pi}{2}\exp\left(-r/\Delta_0\right)\right)$ is available [5].

For the opposite case $R << \Delta_0$, it is possible to use the approximation of isotropic magnet with exact Belavin–Polyakov soliton (BPS) $\tan\left(\theta_0(r)/2\right) = r/R$ [6].

In order to describe magnons on the soliton background, let us introduce small deviations of angular variables θ and ϕ from the static ones,

$$\theta(r,t) = \theta_0(r) + \sum_\alpha f_\alpha(r)\cos\left(m\chi + \omega_\alpha t\right), \quad \phi(r,t) = \phi_0(\chi) + \sin^{-1}\theta_0 \cdot \sum_\alpha g_\alpha(r)\sin\left(m\chi + \omega_\alpha t\right),$$

where $\alpha = (k,m)$ is a full set of eigennumbers. Omitting index α, we then finally obtain the following eigenvalue problem (EVP) for the functions *f* and *g*:

$$\begin{aligned}
&\left[-\nabla_r^2 + \left(\frac{1}{r^2} - \frac{1}{\Delta_0^2}\right)\cos 2\theta_0 - \frac{m^2}{r^2}\right] f = -\left[\frac{\omega}{D} + \frac{2m\cos\theta_0}{r^2}\right] g, \\
&\left[-\nabla_r^2 + \left(\frac{1}{r^2} - \frac{1}{\Delta_0^2}\right)\cos^2\theta_0 - \left(\frac{d\theta_0}{dr}\right)^2 - \frac{m^2}{r^2}\right] g = -\left[\frac{\omega}{D} + \frac{2m\cos\theta_0}{r^2}\right] f.
\end{aligned} \tag{2}$$

We will discuss Dirichlet boundary conditions for the EVP (2)

$$f(R) = 0, \quad g(R) = 0, \tag{3}$$

which corresponds to a fixed value of magnetization at the boundary.

Note that free magnon solutions can be found if we set $\cos\theta_0 = 0$; therefore the resulting modes have a form

$$f(r), g(r) \propto J_{|m|}(kr), \tag{4}$$

J_m is Bessel function with eigennumbers $k_{m,n} = j_{m,n}/R$, where $j_{m,n}$ means the n-th zero of J_m. Therefore, modes with $m = \pm|m|$ have the same frequencies, so there is degeneration for all modes with $m \neq 0$.

3. Normal modes for the BPS

Let us consider the case of BPS, which corresponds to the isotropic magnet $R << \Delta_0$. Using the explicit form for the static solution, we can easily show that the "potentials" in both equations (2) are identical. Due to this unique property of the isotropic model, magnon amplitudes f and g coincide, so it is possible to reduce the system (2) to the 2d–radial Schrödinger EVP,

$$\left[-\nabla_r^2 + \frac{m^2 + 2m\cos\theta_0 + \cos 2\theta_0}{r^2}\right] f_m = k^2 f_m \qquad (5)$$

In the case of $k = 0$ there is an exact solution of EVP (5), $f_m^{(0)}(r) = (r/R)^m \sin\theta_0$ due to the restoration of the conformal invariance of the model [8]. Using the method, which is developed in [9], it is possible to find an exact analytical solution for the translational mode $f_{m=-1}$ for arbitrary wave number k,

$$f_{-1}(r) = J_2(kr) - \frac{2}{kr} \cdot \frac{J_1(kr)}{(r/R)^2 + 1}. \qquad (6)$$

In the infinite system ($R \to \infty$) the lowest eigenvalue $\omega = 0$, which corresponds to the displacement of a soliton as a whole. For the finite system size it is natural to wait for a small value too. Using boundary conditions (3), eigenfrequencies $\omega_{m=-1,n} = D z^2/R^2$ can be found as solutions of algebraic equation $J_1(z)/J_2(z) = z$, with the lowest eigenvalue $\omega_{-1,0} \approx 3.36 D/R^2$.

For another magnon modes solution of EVP (5) can be found numerically only. Interaction with the BPS removes the degeneracy of the magnon spectrum for modes with $m = \pm|m|$, see Fig.1.

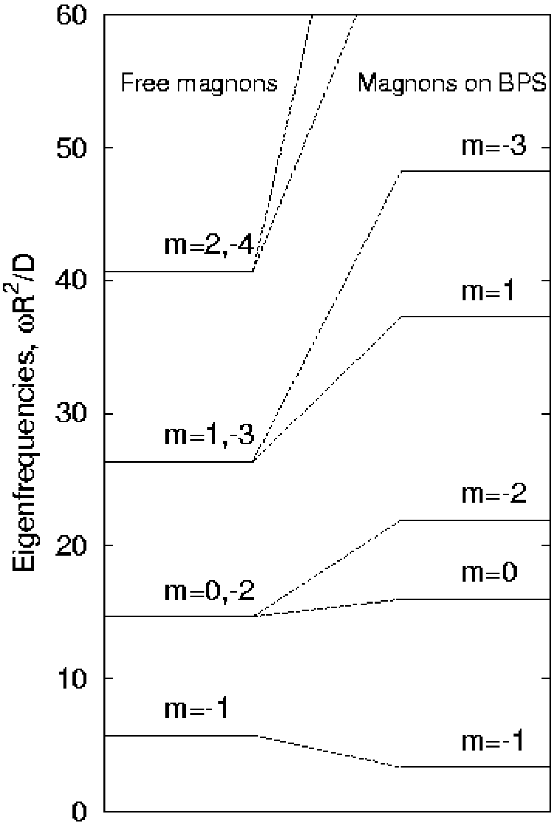

Fig.1. Magnon eigenspectrum for free magnons and for magnons on the BPS background.

4. Normal modes and vortex–magnon scattering for the OPV

Let us consider magnon modes in the easy–plane FM with OPV, which profile has well–pronounced exponential decay, when $R >> \Delta_0$. In this case far from the vortex core there exist quasi–free magnon modes like (4); the vortex–magnon interaction can be taken into account in the main approximation by the scattering term [7],

$$f_m, g_m \propto J_{|m+1|}(kr) + \sigma_m(k\Delta_0) Y_{|m+1|}(kr), \qquad (7)$$

The quantity σ_m determines the scattering amplitude. For partial waves with $m = 0, \pm 1$ it is possible to find an exact solutions for "zero" modes; frequencies of such modes tends to zero for the infinite size of the magnet,

$$f_m^{(0)}(r) = |m| \cdot \frac{d\theta_0(r)}{dr}, \quad g_m^{(0)}(r) = -m \sin\theta_0(r) \cdot r^{-|m|} . \tag{8}$$

The presence of these modes is dictated by the symmetry: the translational mode with $m = \pm 1$ describes the displacements of a soliton as a whole, the rotational one (*m=0*) corresponds to the spin rotation in soliton. Then, starting from the exact solution for "zero" modes $f_m^{(0)}$ and $g_m^{(0)}$, we can construct the solution for small but finite k ($k << 1/\Delta_0$) using a perturbation–theory expansion in respect to k^2. On the other hand, we can explain the results in terms of the scattering problem (7).

Tedious calculations lead to long-wavelength asymptotics of the scattering amplitude,

$$\sigma_{m=\pm 1}(k\Delta_0) = \mp \frac{\pi}{4}(k\Delta_0), \quad \sigma_{m=0}(k\Delta_0) = -\frac{\pi}{2}(k\Delta_0)^2 \cdot \ln\left(1/k\Delta_0 \right), \tag{9}$$

and corresponding asymptotics for partial waves in the region $\Delta_0 << r << 1/k$,

$$g_{m=\pm 1}(r) \propto r \pm \frac{\Delta_0}{kr}, \quad g_{m=0}(r) \propto 1 - \frac{1}{2}(k\Delta_0)^2 \ln^2\left(r/\Delta_0 \right). \tag{10}$$

On the basis of scattering data we are able to calculate normal modes in the FM particle. Using the Dirichlet boundary conditions (3) for the finite size magnet, it is possible to find the eigenfrequencies $\omega_{m,n} \approx c \cdot z_{m,n} / R$ as the solutions of the equation $J_{|m+1|}(z) + \sigma_m (z\Delta_0/R) Y_{|m+1|}(z) = 0$. However, for $m = 0, \pm 1$, the symmetry of the problem is higher (there are zero modes in infinite magnet). Naturally, in the finite size magnet case there are quasi–Goldstone modes with anomalously small frequencies, i.e. *kR*<<1. Applying boundary conditions to the asymptotics (10), one can find eigenfrequencies for such quasi–Goldstone modes in the form

$$\omega_{m=-1} \approx c\Delta_0 / R^2 , \quad \omega_{m=0} \approx c\sqrt{2}/R \ln(R/\Delta_0). \tag{11}$$

For the mode $m = +1$, the value of eigenfrequency has a standard form like $c \cdot z_{-1,n} / R$ as for the rest ones. Therefore, there is a nonsmall splitting for the lowest two modes; for the next modes with $|m| = 1$, it is about $(\Delta_0 / R)^2$; for the rest ones with $|m| > 1$, it is smaller, but exist always.

In conclusion, we would like to remark that for the limit cases ($R << \Delta_0$ and $R >> \Delta_0$) our study demonstrates two features: the splitting of doublets with $m = \pm |m|$, which are degenerated in homogeneous case, and appearance of modes with anomalously small frequencies.

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