Dynamics of Vortices and Their Contribution to the Response Functions of Classical Quasi-Two-Dimensional Easy-Plane Antiferromagnet

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The mechanism of magnetic vortex motion in the classical easy-plane antiferromagnet and the vortex gas contribution to the response functions of such magnets are considered for temperatures above the Kosteritz-Thouless transition. Unlike a ferromagnet, gyrotropical properties of such vortices arise only in sufficiently strong transversal magnetic field. Because of that, the magnetic field produces an important effect on the shape and the width of the central peak of the dynamical structure factor of antiferromagnets.

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Nonlinear elementary excitations of quasi-two-dimensional (2D) spin systems such as magnetic vortices bring about specific solitonical contributions to all the thermodynamical characteristics of a magnet [1]. There is a gas of quasifree magnetic vortices in the isotropic easy-plane magnet above the critical temperature T_c. These solitons make some contribution to the response functions of the magnet and shape the so-called central peak (CP) of the dynamical structure factor (DSF). Such a contribution was calculated in Refs. [2–5] for the vortices in a ferromagnet, in Ref. [6] for in-plane vortices in an antiferromagnet (AFM), and in Ref. [7] for in-plane and out-of-plane vortices in the AFM with very weak anisotropy and asymmetric Dzyaloshinskiĭ interaction (DI). On the other hand, comparison with simulations (see Refs. [6, 7]) proves that dynamics of out-of-plane vortices in the AFM differs strongly from those mentioned above; they must have an effect on rms velocity of vortices and consequently on the vortex contribution in the DSF.

In this paper we have considered the dynamics of the out-of-plane vortex and their ensembles in the AFM with weak easy-plane anisotropy, asymmetric DI, and the external magnetic field, which is perpendicular to the easy-plane, calculated vortex average velocities and their contribution to the DSF. It was proved that the presence of a magnetic field $\mathbf{H}$ by contrast to the DI considerably changes vortex dynamics which substantially transforms the shape and the position of the CP. Besides that we showed that the value of rms velocity is greater than in the ferromagnet [8] for the same values of parameters of magnets and depends critically on the magnetic field. Strong dependence on temperature of vortex gas rms velocity is predicted for slight fields or absence of field.

The model.—Let us consider the two-sublattice model of the AFM. Instead of magnetic moments of sublattices $\mathbf{M}_1$ and $\mathbf{M}_2$, $|\mathbf{M}_1| = |\mathbf{M}_2| = M_0$, it is convenient to introduce the normalized magnetization vector $\mathbf{m}$ and the normalized sublattice magnetization vector $\mathbf{l}$,

$$\mathbf{m} = (\mathbf{M}_1 + \mathbf{M}_2)/2M_0 , \quad \mathbf{l} = (\mathbf{M}_1 - \mathbf{M}_2)/2M_0 ,$$

which are related by

$$\mathbf{m}^2 + \mathbf{l}^2 = 1 \quad (\mathbf{m}, \mathbf{l}) = 0. \tag{1}$$

Supposing $|\mathbf{m}| \ll |\mathbf{l}| \approx 1$ (this assumption is justifiable in a weak magnetic field $H \ll H_e$ and a weak DI, $H_d \ll H_e$, where H_e and H_d are exchange and DI fields, respectively), let us write down the energy density of the AFM [9]

$$W = M_0^2 \left\{ \frac{\delta}{2}\mathbf{m}^2 + \frac{\alpha}{2}(\nabla \mathbf{l})^2 + \frac{\beta}{2} l_z^2 + d\,(\mathbf{e}_z \cdot [\mathbf{m} \times \mathbf{l}]) - 2\mathbf{h} \cdot \mathbf{m} \right\} . \tag{2}$$

Here the unit vector $\mathbf{e}_z$ is directed along the hard axis of the crystal, M_0 is the saturation magnetization, $\delta = H_e/2M_0$ and α are the constants of the uniform and nonuniform exchange, respectively, $\beta > 0$ is the anisotropy constant, $h = H/M_0$, and $d = 2H_d/M_0$ is a constant of the DI.

To investigate the nonlinear dynamics in the AFM let us switch over to the effective equation for $\mathbf{l}$ only on the basis of the generalized σ model of $\mathbf{n}$ field for the sublattice magnetization unit vector $\mathbf{l}$; see Refs. [10, 11] (the equivalent description through the angular variables for $\mathbf{M}_1$ and $\mathbf{M}_2$ was proposed by Mikeska [12] and used in Refs. [5–7]). It is convenient to use angular variables for $\mathbf{l}$, $l_z = \cos\theta$, $l_x + i l_y = \sin\theta \exp(i\varphi)$. Neglecting the dissipation processes, the equations of motion can be obtained from the Lagrangian,

$$L = \frac{\alpha a M_0^2}{2} \int d^2x \left\{ \frac{1}{c^2} \left(\frac{\partial \theta}{\partial t} \right)^2 - (\nabla\theta)^2 - \frac{1}{l_H^2} \cos^2\theta + \sin^2\theta \left[\frac{1}{c^2} \left(\left(\frac{\partial \varphi}{\partial t} \right)^2 - 2gH \frac{\partial \varphi}{\partial t} \right) - (\nabla\varphi)^2 \right] \right\}. \tag{3}$$

The magnetization vector $\mathbf{m}$ can be expressed in terms of $\mathbf{l}$ and $\partial\mathbf{l}/\partial t$ only:

$$\mathbf{m} = \frac{d}{\delta}[\mathbf{l} \times \mathbf{e}_z] + \frac{2}{\delta}\{\mathbf{h} - \mathbf{l}(\mathbf{h}\cdot\mathbf{l})\} + \frac{2}{g\delta M_0}\left[\frac{\partial \mathbf{l}}{\partial t} \times \mathbf{l}\right]. \tag{4}$$

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Here $c = gM_0\sqrt{\alpha\delta}/2$ is the minimum phase velocity of spin waves, g is the gyromagnetic ratio, a is the lattice constant, $l_H = l_0\left(1 + H^2/H_0^2\right)^{-1/2}$ is the characteristic magnetic length, $l_0 = (\alpha/\tilde{\beta})^{1/2}$, $H_0 = H_e(\delta/\tilde{\beta})^{1/2}/4$, and $\tilde{\beta} = \beta(1+d^2/\delta\beta^2)^{1/2}$ is the effective anisotropy constant, renormalized by the DI; see [13, 14].

Note that in the framework of the model (2) the DI was exhibited in the statical characteristics only, namely, in the formula for $\mathbf{m}$ and in the renormalization of the anisotropy constant β. But in fact the additional term $e_{ijk}D_{li}l_il_l\partial l_k/\partial t$ can occur in the Lagrangian (3) for the arbitrary type of the DI [13]; see also Ref. [14]. If $D_{ij} = de_{ijk}(\mathbf{e}_z)_k$ the term reduces to the total derivative and can be omitted.

It should be noted that at $H = 0$ and $D_{ij} = de_{ijk}(\mathbf{e}_z)_k$ the dynamics of the AFM magnetization is Lorentz invariant (LI) with the characteristic velocity c, but when $H \neq 0$ we have the gyrotropical term $\propto gH\sin^2\theta(\partial\varphi/\partial t)$, breaking down LI properties [in the ferromagnet we have an opposite situation in which the dynamical term of the Lagrangian $\propto (1-\cos\theta)(\partial\varphi\partial t)$ is of a gyrotropical nature only, but terms with $(\partial\theta/\partial t)^2$ and $(\partial\varphi/\partial t)^2$ are absent].

In the dissipationless limit, the system has such integrals of motion as magnet energy E and momentum of magnetization field $\mathbf{P}$. The expression for the momentum may be obtained from the Lagrangian (3): $\mathbf{P} = \mathbf{P}_{\rm LI}+\mathbf{P}_g$,

$$\mathbf{P}_{\rm LI} = -\frac{\alpha a M_0^2}{c^2}\int d^2x\left[\nabla\theta\frac{\partial\theta}{\partial t} + \nabla\varphi\frac{\partial\varphi}{\partial t}\sin^2\theta\right],$$

$$\mathbf{P}_g = \frac{\alpha a M_0^2}{c^2}\int d^2x \sin^2\theta gH\nabla\varphi\,, \quad (5)$$

where the term $\mathbf{P}_{\rm LI}$ is a customary LI one, and the gyrotropical term $\mathbf{P}_g$ is caused by the presence of the magnetic field. This expression has no singularities connected with the undifferentiability of φ when $r\to 0$ and $\theta\to 0$ or π (the problem of the undifferentiability of φ was discussed by Papanicolaou and Tomaras for the vortices of a ferromagnet; see Ref. [15]).

It is of interest to note that in the case of a stationary nonuniform state of the AFM such as $\theta = \pi/2$, $\varphi = \mathbf{k}\cdot\mathbf{r}$, the presence of the term $\mathbf{P}_g$ leads to the nonzero momentum $\mathbf{P} = \mathbf{k}gH\alpha a M_0^2 S/c^2$, where S is the AFM area. Such behavior is typical for a superfluid liquid, which is described by a complex order parameter $\Psi = |\psi|\exp(i\varphi)$. The momentum density of the superfluid flow is determined by the well-known expression $\mathbf{p} = |\psi|^2\nabla\varphi = \rho_s\mathbf{v}_s$, where ρ_s is the density of the superfluid component and $\mathbf{v}_s$ is its velocity. The similarity of these expressions makes it possible to talk about a fundamental analogy between superfluid systems and easy-plane magnets (this problem was discussed for the case of ferromagnets; see Refs. [16, 17]). The momentum density can be naturally juxtaposed with the quantity $\rho_s\mathbf{v}_s$, while the energy density corresponds to the quantity $\rho_s\mathbf{v}_s^2/2$. It follows from the above formulas that the quantity $\rho_s^{\rm AFM} = 2\alpha a M_0^2(gH/c^2)^2 = 8aH^2/c^2\delta$ can be treated as the analog of the superfluid density ρ_s for the dynamics of the easy-plane AFM.

Vortex dynamics.—The structure of the vortex is determined by equations for θ and φ following from (3). For the motionless vortex the solution has the form

$$\varphi = \varphi_0 + \nu\chi\,, \quad \theta = \theta(\xi)\,, \quad \xi \equiv r/l_H,$$

both in the ferromagnet and in the AFM, where $\varphi_0 =$ const, χ and r are the polar coordinates in the magnet plane xy, and $\nu = \pm1, \pm2, \ldots$ determines the vortex topological charge (vorticity). The function $\theta(\xi)$ is a solution of the ordinary differential equation

$$\frac{d^2\theta}{d\xi^2} + \frac{1}{\xi}\frac{d\theta}{d\xi} = \sin\theta\cos\theta\left(1 - \frac{\nu^2}{\xi^2}\right),$$

with boundary conditions $\theta(0) = \pi(1-p)/2$ and $\theta(\infty) = \pi/2$, where $p = \pm1$ determines the second topological charge of the vortex (polarization) [17]. The energy of the static vortex diverges as the logarithm of the area S of the vortex, and when $|\nu| = 1$ it is determined by the expression $E_0 = \frac{1}{2}\pi\alpha a M_0^2\ln(5.67S/l_H^2)$; see Ref. [9].

The basic distinctions of vortex dynamics in the AFM from the case of the ferromagnet are explained by the study of its dynamical properties. It was mentioned above that the Lagrangian (3) has the LI property when $H = 0$, and the distribution for the vector $\mathbf{l}$ of the vortex moving with the velocity $\mathbf{v} = v\mathbf{e}_x$ can be obtained from the static solution by the Lorentz transform, $x \to x' = (x - vt)(1 - v^2/c^2)^{-1/2}$, $y \to y$. The energy and momentum of the vortex when $H = 0$ are determined by the LI formulas, $E_{\rm LI}(v) = E_0(1 - v^2/c^2)^{-1/2}$, $\mathbf{P}_{\rm LI}(v) = (\mathbf{v}/c^2)E_{\rm LI}(v)$, where E_0 is the energy of the motionless soliton. Thus, the vortex effective mass in the case where $H = 0$ is proportional to $\ln S$.

In the presence of the magnetic field, the examination of the vortex motion is more complex. In particular, there is no exact solution describing the moving vortex in the ferromagnet. Unlike this case, we were able to construct the exact solution for the vortex in the AFM moving with constant velocity $v < c$ and $H \neq 0$,

$$\varphi = \varphi_0 + \arctan\left(\frac{y}{x'}\right) + \mathbf{k}\cdot\mathbf{r}\,, \quad \mathbf{k} = \mathbf{v}gH/c^2\,,$$

$$\theta = \theta(\xi')\,, \quad \xi' \equiv r'\left(l_H^{-2} - k^2\right)^{1/2}, \quad r'^2 \equiv x'^2 + y^2. \quad (6)$$

It is easy to express the vector $\mathbf{m}$ in the terms of angular coordinates,

$$m_x + im_y = -\sin\theta\left(i\frac{d}{\delta} + \frac{2h}{\delta}\cos\theta\right)e^{i\varphi} + \frac{2}{g\delta M_0}\left(i\frac{\partial\theta}{\partial t} + \sin\theta\cos\theta\frac{\partial\varphi}{\partial t}\right)e^{i\varphi}\,,$$

$$m_z = \sin^2\theta\left(\frac{2h}{\delta} - \frac{2}{g\delta M_0}\frac{\partial\varphi}{\partial t}\right). \quad (7)$$

The term with $\mathbf{k}$ is caused by the vortex "freezing into the condensate." The effect of "freezing into" exists both in ferromagnets and AFMs (when $H \neq 0$), and it makes possible the vortex motion only with hydrodynamical fluxes. Therefore we can omit the inertial term in the equation of motion. The energy of such a soliton $E(v) = E_{\rm LI}(v) + \mu v^2/2$, where $\mu = \rho_s^{\rm AFM} S$ is the condensate mass.

To investigate the dynamical properties of the AFM vortex ensemble, we use an approach [18] based on an analysis of the expression for the magnet momentum $\mathbf{P}$ and force balance conditions $d\mathbf{P}/dt = \mathbf{F}$, where $\mathbf{F}$ is the external force acting on the vortex. For a steadily moving soliton $\theta = \theta(\mathbf{r} - \mathbf{q}(t))$ and $\varphi = \varphi(\mathbf{r} - \mathbf{q}(t))$, where $\mathbf{q}(t)$ describes the motion of the vortex center. In accordance with (5), the momentum contains the two terms $\mathbf{P}_{\rm LI}$ and $\mathbf{P}_g$. As in the case of the ferromagnet, $\mathbf{P}_g$ contains the term $\mathbf{P}_g^{(0)}$, which is finite when $\mathbf{v} \to 0$. The value

$$\frac{d\mathbf{P}_g^{(0)}}{dt} = \frac{\alpha a M_0^2 g H}{c^2} \int d^2x \sin 2\theta \left(\nabla\varphi \frac{\partial \theta}{\partial t} - \nabla\theta \frac{\partial \varphi}{\partial t} \right)$$

can be transformed to $d\mathbf{P}_g^{(0)}/dt = -G\,[\mathbf{v} \times \mathbf{e}_z]$, where

$$G = -2\pi\nu\alpha a M_0^2 g H/c^2 \,. \tag{8}$$

When writing down the force balance condition as $G\,[\mathbf{v} \times \mathbf{e}_z] + \mathbf{F} = 0$, the term with G may be interpreted as some gyrotropical force acting on the moving vortex. Such gyroforce is always present in the case of the ferromagnet and determines the most important properties of the dynamics of vortices and their ensembles, in particular, the value of rms $\overline{u}$; see Ref. [8] and below in the text. The gyroforce in the AFM is nonzero only at $H \neq 0$, and for the same values of the parameters M_0 and ν, it is less than that in the ferromagnet [absolute value $G_{\rm AFM} \approx (8H/H_e)\, G_{\rm FM}$, the order of the magnitude of H_e is 100–1000 kOe]. Let us point out that it is only in the case of the AFM that G does not depend on the second topological charge p.

On the basis of the previously obtained relations, let us write down the effective equation of motion for the ensemble of vortices,

$$G \left[\frac{\partial \mathbf{q}_i}{\partial t} \times \mathbf{e}_z \right] + \mathbf{F}_{e,i} - \eta \frac{\partial \mathbf{q}_i}{\partial t} = 0 \,. \tag{9}$$

Here $\mathbf{q}_i$ is the ith vortex-center coordinate and G is the above-determined gyrotropical constant. The meaning of the remaining terms is the same as in the ferromagnet: $\mathbf{F}_{e,i} = -\nabla_i \mathcal{H}_{\rm int}$ describes the interaction between vortices, Hamiltonian $\mathcal{H}_{\rm int} = -2 \sum_{i \neq j} e_i e_j \ln |\mathbf{q}_i - \mathbf{q}_j|$ is typical for 2D Coulomb interaction, "electrical charge" $e_i = \nu_i M_0 \sqrt{\pi \alpha a}$, and η is a viscous coefficient; cf. [19].

Vortex gas average velocity.—Equation (9) was used by Huber [8] in the thermodynamical calculation of vortex gas velocity in the ferromagnet. The features of vortex gas motion are of substantial interest in the calculation of solitonic response functions. Investigations of the case of the ferromagnet by Mertens *et al.* [2] showed that vortices produce an essential contribution to the CP region. Thermodynamical characteristics of vortex gas in the AFM exhibit salient features connected with the vanishing of G as $H \to 0$.

Let us introduce the self-consistent effective "electric field" $\mathbf{E}$, describing interaction with other vortices. There is a formal similarity between the equation of vortex motion and the equation of motion of guiding center in a 2D plasma, which lies in a perpendicular magnetic field [8]. It allows us to estimate the value of $\langle E^2 \rangle$ by results obtained by Taylor and McNamara [20]. $\langle E^2 \rangle = n_v \pi e^2 \ln \Lambda$, where n_v is the equilibrium vortex density, and Λ determines the proximity to the thermal equilibrium. For the cases of a random and a thermal distribution Λ is expressed by $\Lambda_R = S/\pi a^2$ and by $\Lambda_T = 4\pi^2 T_c / n_v e^2 a^2$, respectively.

Using this $\langle E^2 \rangle$ and Eq. (9) we obtain the value of the rms vortex velocity

$$\overline{u} = \frac{e \langle E^2 \rangle^{1/2}}{(G^2 + \eta^2)^{1/2}} = \frac{c}{2} \left(\frac{\pi n_v l_0^2 H_0^2}{H^2 + H_r^2} \ln \Lambda \right)^{1/2} , \tag{10}$$

where for convenience we use the typical fields H_0 (see above) and $H_r = \eta(g\delta/8\pi a)$; H_r is proportional to the relaxation coefficient. Estimating η as in Ref. [19], we obtain $H_r \approx 0.05 H_0$ when $T \approx T_c$. Thus, for the value $\overline{u}/\overline{u}_H \approx H_e/4H$ is seen to be inversely proportional to H, but for $H \ll H_r$, $\overline{u}/\overline{u}_H$ does not depend on H. Rather, $\overline{u}/\overline{u}_H \approx H_e/4H_r$, where

$$\overline{u}_H = \frac{g M_0}{2} (\alpha \tilde{\beta})^{1/2} \left(\pi n_v l_0^2 \ln \Lambda \right)^{1/2}$$

is a typical rms velocity in the ferromagnet obtained by Huber [8]. Therefore the rms velocity in the AFM is greater than that in the ferromagnet. These results agree in kind with data simulated by Völkel *et al.* [6]. It would be interesting to check the dependence $\overline{u} \propto 1/H$, but numerical simulations for $H \neq 0$ have not been carried out, as far as we know. In the case where $H < H_r$ the value $\overline{u}$ does not depend on H, but it is inversely proportional to relaxation constant η. Since $\eta \propto T^n$, $n = 2$ for the ferromagnet [19] and $n = 3$ for the AFM [21], this formula describes a decrease of the value of $\overline{u}$ with increasing T stronger than in the ferromagnet. Note that the computer simulations demonstrate a decrease of $\overline{u}$ for the AFM [7] in a small vicinity of T_c when T increases stronger than for the ferromagnet [6].

DSF calculation.—Now we proceed to the calculation of the vortex contribution to the correlation functions. Because of (1) contributions of terms $\mathbf{m}$ and $\mathbf{l}$ are independent and additive. Moreover, they give correlations at two different positions in $\mathbf{q}$ space. Namely, $\mathbf{l}$ determines components of the CP, which are centered about the AFM Bragg peak, i.e., at the position $\mathbf{K}^0 = (\pi, \pi)$; cf. Ref. [6]. But $\mathbf{m}$ makes a contribution at $\mathbf{q} = 0$.

The concrete calculation can be made in analogy with the case of the ferromagnet [2], so we are not going to detail it here. Let us note that the CP for the out-of-plane correlations has a Gaussian shape, and for in-plane ones it has a (squared) Lorentzian shape. Both for in-plane and for out-of-plane correlations, the CP widths $\Delta\Gamma_z = q\overline{u}$ and $\Delta\Gamma_x \approx 1.14\overline{u}n_v^{1/2}$ increase strongly as the field diminishes; see Eq. (10).

It is important to note that the magnetic field and the DI affect different components of the DSF and can be checked independently. The form of the in-plane components is not a function of the magnetic field. It is determined by such expressions as in [6] and [7], which take into account the DI. For the out-of-plane correlations the DI makes no contribution, but the form of the DSF components depends substantially on the value of the magnetic field:

$$S^{zz}(\mathbf{q},\omega) = |f_1(\mathbf{q})|^2 F_G(\mathbf{K}^0 - \mathbf{q},\omega) + \frac{4h^2}{\delta^2} n_v \delta(\mathbf{q})\delta(\omega) + \frac{4h^2}{\delta^2}|f_2(\mathbf{q})|^2 F_G(\mathbf{q},\omega).$$

Here $F_G(\mathbf{q},\omega)$ determines the well-known [2] expression for the the Gaussian CP,

$$F_G = \frac{n_v}{2\pi^{3/2} q\overline{u}} \exp\left[-(\omega/q\overline{u})^2\right],$$

$f_k(\mathbf{q}) = \int d^2x \cos^k\theta(r)\exp(i\mathbf{q}\cdot\mathbf{r})$, $k = 1,2$, are two different vortex form factors, which determine distributions of $\mathbf{l}$ and $\mathbf{m}$, respectively. Let us note that the intensity of the third term $\propto H^2$. The analysis of this dependence can be a good test for comparison of experimental and theoretical data.

Thus our investigation demonstrates that vortex dynamics in the AFM differs substantially from that in the ferromagnet and besides the magnetic field, which is perpendicular to the easy plane, strongly affects it. The vortex dynamics is described by the LI equations when $H = 0$. In the presence of the field, effects of vortex "freezing into the condensate" and of gyrotropical motion appear. There is a transfer from the viscous motion to the gyrotropical one when H increases. For all reasonable fields $H \ll H_e$, which do not destroy the AFM order, the rms velocity $\overline{u}$ and the CP widths $\Delta\Gamma \propto \overline{u}$ are greater than Huber value: $\overline{u} \propto (H_e/H)u_H$ for $H \gg H_r$ and $\overline{u} \propto (H_e/H_r)u_H$ for $H \ll H_r$, where $H_r \approx 0.05H_0 \ll H_0 \ll H_e$. Moreover, the presence of field gives rise to the particular contribution to the out-of-plane DSF components whose intensities are substantially dependent on the field.

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