Magnetic vortices in a highly anisotropic ferromagnet with unit sp

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It is shown that in a ferromagnet with unit spin, in which the average value of spin is small in view of its quantum contraction, the dynamics of a two-dimensional soliton (vortex) is quite different from that in a Heisenberg magnet. The vortex freezing and gyroeffects emerge only in a magnetic field, while the dynamics is Lorentz invariant in the absence of a field.

It is well known that topological solitons of nonlocalized vortex type (disclinations) play the role of elementary excitations in two-dimensional (2*D*) easy-plane magnets. Vortices determine the violation of the long-range order of the Berezinskii–Kosterlitz–Thouless type transition[1,2] and lead to the emergence of the central peak in correlation functions.[3-11] The central peak effects were observed experimentally in the magnets $BaNi_2(PO_4)_2$ [Ref. 6] and $CoCl_2$ [Ref. 7], were described theoretically both analytically[4,8] and by using numerical simulation[9,11] techniques. The analytic description was based on vortex solutions obtained for weakly anisotropic Heisenberg magnets, and the anisotropy energy in (real and numerical) experiments could be comparable with the exchange interaction energy.

The static, and especially dynamic, properties of magnets with spin $s = 1$ and a strong one-ion anisotropy may differ significantly from the properties of Heisenberg magnets. For example, strongly anisotropic magnets may acquire nonmagnetic (quadrupole) phases with zero average spin, additional magnon branches,[12,13] solitons (domain boundaries and vortices) with a change in magnetization along the length.[14-16] Among other things, purely two-dimensional vortex solutions exist in the case of a ferromagnet (FM) with $s = 1$ and strong effects of quantum suppression of spin. In these solutions, the spins do not leave the easy plane and the magnitude of magnetization at the center of the vortex becomes equal to zero.[14] We shall study the structure of such vortices and their dynamic properties in an FM in the vicinity of the transition from ordered to quadropole phase, when the role of quantum contraction of spin is the most significant.

The dynamics of magnetization in a highly anisotropic plane ferromagnet (PFM) may be described by using the Lagrangian for the components of average spin projection on the easy plane xy. For the model of a purely uniaxial PFM near the phase transition from the magnetic phase to the quadrupole phase, this Lagrangian was obtained in Ref. 16. Using the parametrization $s_x = s\cos\varphi$, $s_y = s\sin\varphi$, we can present this Lagrangian in the form

$$L = \frac{1}{8} IR_0^2 \left\{ \frac{\dot{s}^2}{c^2} - (\nabla s)^2 + s^2 \left[\frac{1}{c^2} (\dot{\varphi}^2 - 2\omega_H \dot{\varphi}) - (\nabla\varphi)^2 \right] - \frac{4}{R_0^2} (s^2 - s_0^2)^2 \right\}, \quad (1)$$

where the dot indicates differentiation with respect to time, $I = I_0/a^3$; a is the separation between spins, I_0 the exchange integral, R_0 the characteristic scale of length, S_0 the equilibrium value of the average spin (in the case of proximity to the phase transition considered by us, $S_0 << 1$), $c = IR_0/\hbar$ the characteristic velocity, $\omega_H = 2\mu_0 H/\hbar$, and μ_0 the Bohr magneton.

As compared in Ref. 16, the above equation also tains an additional term which is linear in $\dot{\varphi}$ and is due to presence of the magnetic field **H** along the difficult axi The emergence of such a term is associated with the existe of an invariant ($\mathbf{H}\,[S, \dot{S}]$) which is nonzero even for a PF The coefficient of this term can easily be determined by us the method developed in Ref. 17.

Using the Lagrangian (1), we obtain the following pression for the integrals of motion:
energy

$$E = \frac{1}{8} aIR_0^2 \int d^2x \left\{ (\nabla s)^2 + \frac{1}{c^2}\dot{s}^2 + s^2 \left[(\nabla\varphi)^2 + \frac{1}{c^2}\dot{\varphi}^2 \right] + \frac{4}{R_0^2}(s^2 - s_0^2)^2 \right.$$

momentum

$$\mathbf{P} = -\frac{1}{4c^2} aIR_0^2 \int d^2x \left\{ \dot{s}\nabla s + s^2(\dot{\varphi} - \omega_H)\nabla\varphi \right\}.$$

Here, a is the lattice constant and has been added to preser the dimensionality.

A stationary two-dimensional topological vortex has th solution

$$\varphi^{(0)} = \varphi_0 + \nu\chi\,, \quad s^{(0)} = s_0 f(r\,2s_0\sqrt{2}/R_0)\,,$$

where r and χ are polar coordinates in the easy plane, φ_0 is constant, and ν an integer describing the topological charge o the vortex. The function $f(\xi)$ satisfies the Gross–Pitaevski differential equation for the density of the superfluid compo nent of a weakly nonideal Bose gas in the vortex[18] (see also Ref. 19). The asymptotic behavior of this function near th vortex axis ($\xi \to 0$) is $f \propto \xi^{|\nu|}$, while away from the axi ($\xi >> 1$) we have $f \simeq 1 - \nu^2/2\xi^2$.

The energy of a static vortex depends logarithmically on the area S of the magnet:

$$E_0 = \frac{1}{8}\pi aIR_0^2 s_0^2 \ln(AS/R_0^2)\,, \quad (5)$$

where A is a quantity of the order of unity, depending on the topological charge ν.

Unlike the static properties of vortices in a PFM, their dynamics differs basically from the vortex dynamics in a superfluid Bose gas as well as in a Heisenberg FM. The nature of the dynamics depends essentially on the presence of a magnetic field H.

If $H = 0$, the dynamics of magnetization is Lorentz invariant. In this case, the "freezing" and dyroeffects typical of vortices in an FM or a Bose gas are not observed. The

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tion corresponding to a steady-state vortex moving with a ity $\mathbf{v}$ along the x-axis is given by the standard Lorentz formation

$$x \to x' = \gamma(x - vt),\ t \to t' = \gamma(t - xv/c^2),$$
$$r \to r' = (x'^2 + y^2)^{1/2},\ \gamma = (1 - v^2/c^2)^{-1/2}.$$

In order to analyze a moving vortex in a field, we ob- e that in terms of the variables S and $\psi = \varphi - \omega_H t$, the em (1) is Lorentz-invariant (cf. Ref. 20). Proceeding from atic solution (4), we arrive at the following description moving vortex:

$$s = s^{(0)}(r'),\ \varphi = \varphi_0 + \nu \tan^{-1}(y/x') + \omega_H(t - t'), \quad (6)$$

re $S^{(0)}(r)$ describes a stationary vortex.

In order to find the physical meaning of the solution (6), compare the PFM dynamics with those of a superfluid id (for the sake of convenience in comparison, we confine selves to the case $v << c$). The condensate phase serves he analog of the angular variable φ in the superfluid tem. The momentum density $aIs_0^2\omega_H\nabla\varphi/4c^2$ can be natu- lly juxtaposed with the quantity $\rho_s\mathbf{v}_s$, while the energy nsity $aIs_0^2R_0^2(\nabla\varphi)^2/8$, corresponds to the quantity $\rho_s\mathbf{v}_s^2/2$ here ρ_s is the density of the superfluid component, and $\mathbf{v}_s$ velocity. It follows from the above formulas that the quan-

$$\rho_s^{(F)} = aIR_0^2s_0^2(\omega_H)^2/4c^4. \quad (7)$$

be treated as the analog of the superfluid density ρ_s for magnetization dynamics of a PFM.

In this case, it can be easily verified that the local veloc- of the "superfluid" motion of a "condensate" in a PFM is $= \nu c^2 e_\chi/r\omega_H + v$, ($\mathbf{v}$ is the vortex velocity). The quantity has the usual asymptotic form of the type e_χ/r near the ex, while it is equal to the vortex velocity $\mathbf{v}$ away from vortex. This points uniquely towards the freezing of the ex in the condensate. For $H = 0$, the superfluid compo- nt is missing in view of (7), and the freezing effects are not erved.

This analogy can be confirmed by direct calculations. ced, the dependence of integrals of motion on the vortex ocity can be represented in the form

$$E = \left[E_0 + (\gamma - 1)\mu c^2\right]\gamma, \quad (8)$$
$$\mathbf{P} = \mathbf{P}_0 + (\gamma\mathbf{v}/c^2)(E_0 + \gamma\mu c^2),$$

ith logarithmic accuracy. Here we have used the notation $\mu = \rho_s^{(F)}S$; S being the area of the magnet. This quantity μ, hich emerges only for $H \neq 0$ and during the violation of orentz invariance, has the meaning of the effective mass of uperfluid condensate in a PFM entrained by a vortex dur- g its motion.

The term P_0 appearing in (8) is also nonzero only for H 0. It is convenient to write the expression for dP_0/dt, since the limit of low velocities this expression is proportional to e vortex velocity:

$$\frac{d\mathbf{P}_0}{dt} = G[\mathbf{v}, \mathbf{e}_z],\ G = \frac{\pi a}{2c^2} IR_0^2s_0^2\omega_H = \pi a\hbar\mu_0 H/I.$$

Such terms are well known for vortices in a Heisenberg FM (see Refs. 3 and 21). The quantity $-d\mathbf{P}/dt$ can be compared with the gyroscopic force acting on a moving vortex (see Ref. 21). It can be seen that in contrast to a Heisenberg FM, the quantity $G \propto H$ in a PFM, and the gyroscopic force effects exist only for $H \neq 0$.

Finally, let us describe the basic laws of vortex motion in a PFM. For $H = 0$ in the nondissipative approximation, a vortex can move freely with a constant velocity [see (6)]. This conclusion is in contradiction with the statement made in Ref. 14. In a quite strong field H, however, the motion of vortices in *a* PFM is the same as in a Heisenberg FM: a vortex moves only with the condensate and its motion is determined by the external force $\mathbf{F}_e$ as well as by the gyroscopic force $\mathbf{F}_g = G[e_z, v]$. The laws governing this motion and the effect of the corresponding forces on the nature of the central peak in the dynamic structural factor were discussed in several works (see Refs. 3 and 4). Note that the conclusions about the nature of vortex dynamics for $H = 0$ and $H \neq 0$ do not contradict the general criterion formulated by Nikiforov and Sonin[22] for the existence of gyroscopic forces.

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NOTATION

Here I is the exchange integral, s_0 the average spin of a lattice site, H the magnetic field, E the energy of a magnetic vortex, P the total momentum of a magnetic vortex, R_0 the characteristic localization radius, ν the topological charge, and c the velocity of a vortex.

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