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Fine Structure of the Spectra of Magnetic Particles in the Vortex State and Their Ordered Arrays

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Abstract—A low-frequency (the frequency ω_{VPM} is in the sub-GHz range) mode exist for submicron magnetic particles in the vortex state. This mode corresponds to oscillations of the vortex center. The other modes form doublets with higher (several gigahertz) frequency and small (about ω_{VPM}) splitting. Collective oscillations exist for lattices of such particles. The dependence of the frequency of these modes on the quasi-momentum can be nonanalytic.

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In recent years, systematic study and practical implementation of purely artificial magnetic materials have begun. Among these, two-dimensional superstructures in the form of lattices of submicron circular magnetic particles (magnetic dots) on nonmagnetic substrates are of special interest (see review [1]). Such systems have interesting physical properties, differing them from conventional objects of physics of small magnetic particles, for example, granular magnets. The existence of inhomogeneous vortex states with a closed magnetic flux for an individual particle should be noted. A specific feature of a particle array is the presence of a regular, purely two-dimensional lattice structure and long-range dipole interaction between particles; this implementation of dipole magnets is convenient for carrying out experiments.

The model of an easy-plane magnet is the simplest one; it allows existence of vortices and analysis of their dynamics [2–4]. For a vortex, the magnetization distribution has the form $M_Z = M_S\cos\theta$, where $\theta = \theta(r)$, the function $\cos\theta(r)$ is localized in the region of the vortex core, $M_Y - iM_X = M_S\sin\theta\exp(i\chi)$, M_S is the saturation magnetization, the z axis is perpendicular to the vortex plane, and r and χ are polar coordinates (Fig. 1). It is convenient to express small magnetization oscillations against the vortex background in terms of the projections on local axes, m_1 and m_2. These projections are described by the equation containing the Schrödinger operator $\hat{H}$ for a charged particle in a magnetic field:

$$i(\partial\psi/\partial t) = \hat{H}\psi + \hat{W}\psi^*, \quad \hat{H} = (-i\nabla - \mathbf{A})^2 + U, \quad (1)$$

where $\psi = m_1 + im_2$, $W(r)$ and $U(r)$ are localized functions, $\mathbf{A} = -(\cos\theta/r)\mathbf{e}_\chi$ is the vector potential, and the effective magnetic field $\mathbf{B} = \mathrm{rot}\mathbf{A}$ is parallel to $\mathbf{e}_z$. The field flux $\mathbf{B}$ is determined by the topological charge of the magnetization field [3, 4]. The magnon eigenmodes have the form

$$\psi_\alpha = u_\alpha(r)\exp(i\Phi_\alpha) + \upsilon_\alpha(r)\exp(-i\Phi_\alpha), \quad \Phi_\alpha = m\chi - \omega_\alpha t, \quad (2)$$

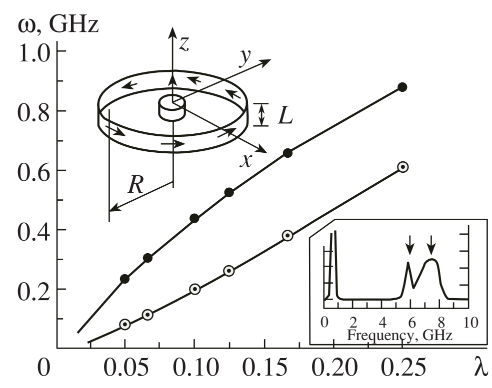

Fig. 1. Lower mode frequency ω_{VPM} ($m = 1$) (filled circles) and frequency splitting $\Delta\omega$ of the first doublet with $|m| = 1$ (open circles) as functions of the shape parameter $\lambda = L/R$. The top left inset schematically shows the vortex state of a disk-shaped particle. The bottom right inset shows the Fourier components of the vortex coordinates for a Permalloy disk with $R = 250$ nm and $L = 60$ nm, measured by high-time-resolution Kerr microscopy [7]; the low-frequency mode and the first doublet can be seen.

Here, the subscript $\alpha = n, m$ includes the number of nodes of the function $u_\alpha(r)$ and the azimuthal number m. Magnons against the background of a vortex are states with a certain orbital momentum, and the effects of the magnetic field for magnons can be clearly represented as the Zeeman splitting of doublets with $m = \pm|m|$. The splitting is not small for $|m| = 1$; hence, the frequency ω_{VPM} of the lowest mode with $m = 1$ is fairly small. This mode describes the vortex center precession with the frequency ω_{VPM}.

For submicron particles of soft magnetic materials, the topological considerations hold true; however, to calculate the frequencies, it is necessary to take into account the nonlocal magnetostatic interaction, which leads to integral equations instead of (1). For a thin particle with a radius R and thickness $L \ll R$, the contributions of the magnetic poles on all surfaces (top, bottom, and lateral) are reduced to the local form [5] and to the model of an easy-plane magnet with fixed boundary conditions, which was considered previously in [2–4].

Only the contribution div**M** of bulk magnetic charges remains nonlocal. Calculation within the perturbation theory [6] shows a universal dependence of frequencies on the ratio $\lambda = L/R$. The fundamental frequencies ($\omega_{n,m} \propto \omega_M\sqrt{L/R} \ll \omega_M$, $\omega_M = 4\pi\gamma M_S$) lie in the range of several GHz, whereas the oscillation frequency of the vortex core and the doublet splitting are small: ω_{VPM}, $\Delta\omega_m \propto \omega_M(L/R) \ll \omega_{n,m}$ (less than 1 GHz) [6]. This result is in agreement with the experimental data for Permalloy particles ($\omega_M = 30$ GHz) [7]. Thus, the eigenmodes for an individual vortex particle radically differ from those for a particle in the homogeneous state.

In the currently prepared lattices, particles are regularly arranged and have modes of collective oscillations with a certain value of the quasi-momentum $\mathbf{k}$. The collective modes of such systems have been investigated for two cases: the ferromagnetic state of an array, which corresponds to the parallel orientation of magnetizations in the core of all particles, and the checkerboard antiferromagnetic state. Spectra of collective oscillations consists of modes with different values of m and n. The modes with $m = 0$ and ±1 are related to the oscillations of the total magnetic moment of a particle. These modes are characterized by dipole coupling and nonanalytical dependence of the frequency on the wave vector: $\omega - \omega_0 \propto |\mathbf{k}|$ at $\mathbf{k} \to 0$ [8]. Such a dependence leads to a change in the peculiarities of the density of states, which is proportional to $|\omega - \omega_0|$. Within the spectrum or near the upper band edge, the singularities of the density of states are conventional for the two-dimensional case: jump or logarithmic divergence (Fig. 2). For the other modes, the interaction corresponds to the magnetic quadrupole one; it decreases more rapidly, and the dependence $\omega(\mathbf{k})$ at small $\mathbf{k}$ is analytic [8]. For the checkerboard antiferromagnetic phase, the density of states at the center of the lower band has a conventional jump; however, for the upper band, there is a singularity of the $|\omega - \omega_0|$ type (Fig. 2).

Near the array boundary, there are specific localized states, which resemble surface (Tamm) states of quasiparticles in crystals. For these modes, $\psi \sim \exp[ikl_\parallel - pl_\perp]$; the quantity k has a meaning of the quasi-momentum lying in the surface plane and $p = p(k)$ has a positive real part, where integers $l_\parallel$ and $l_\perp$ enumerate the nodes in the surface plane and along the normal to the surface, respectively. For the ferromagnetic and checkerboard antiferromagnetic phases of the array, the frequencies of these modes are above and below the main band, respectively.

The collective modes determine the stability of different phases of the lattice of magnetic particles. The ferromagnetic state of the array is stable only in the presence of a sufficiently strong field: $H > H_0 = 9.04H^*$, where $H^* = m_0/a^3$, m_0 is the magnetic moment of an individual particle, and a is the lattice constant [9]. With a decrease in field, the lower boundary of the spectrum of collective oscillations decreases (Fig. 2) and, at $H \to H_0$, the ferromagnetic phase is unstable due to the oscillations with small $\mathbf{k}$. In the checkerboard antiferromagnetic state, which is stable at $H < H_1 = 2.644H^*$ [9], the spectrum consists of two bands; with

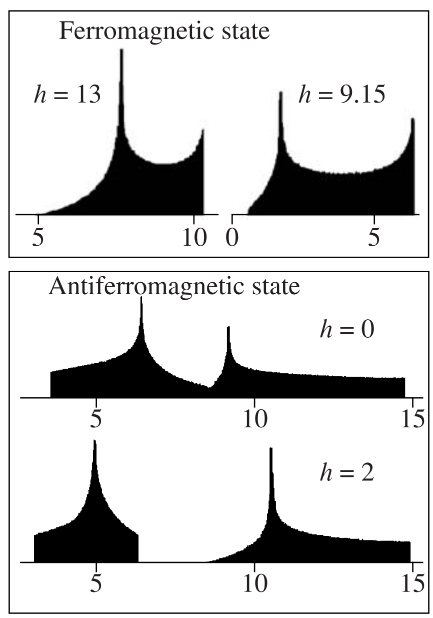

Fig. 2. Density of states of collective oscillations of magnetic moments for a lattice of magnetic particles (in arbitrary units) as a function of frequency (in γH^* units) in the cases of the (a) ferromagnetic and (b) checkerboard antiferromagnetic ordering of magnetic moments in the vortex cores at different magnetic fields. The magnitudes of the magnetic field $h = H/H^*$ are indicated near the corresponding plots.

an increase in the magnetic field, a gap is formed between them. The magnetic field weakly affects the upper and lower boundaries of the spectrum (Fig. 2), and the instability of the checkerboard antiferromagnetic state is due to the mode localized near the array boundary; this conclusion is in agreement with the data of [10].

The doublet structure of the modes of an individual particle is naturally transformed into the fine structure of the modes of collective oscillations. The character of collective modes is determined by the relation between two small parameters: doublet splitting $\Delta\omega_m$ and the frequency shift ω_{int} due to the interaction. If the interaction is weak ($\omega_{int} < \Delta\omega_m$), the spectrum of collective modes includes the nearby branches of oscillations with certain values of the azimuthal number, $m = +|m|$ and $m = -|m|$, i.e., the "angular traveling waves" in the form $\psi^{(+)} \sim \exp(i|m|\chi - i\omega t)$ and $\psi^{(-)} \sim \exp(-i|m|\chi - i\omega t)$. In the other limit ($\omega_{int} > \Delta\omega_m$), the fact that the lattice symmetry is lower than the radial symmetry (characteristic of an individual particle) manifests itself. In this case, the doublet structure is retained, but the splitting is now determined by the value ω_{int}, and the doublet components are combinations of states with $m = \pm|m|$, i.e., "angular standing waves" $\sin(|m|\chi + \omega t)$ and $\cos(|m|\chi + \omega t)$.

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