Switching between different vortex states in two-dimensional easy-plane magnets due to an ac magnetic field

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Using a discrete model of two-dimensional easy-plane classical ferromagnets, we propose that a rotating magnetic field in the easy plane can switch a vortex from one polarization to the opposite one if the amplitude exceeds a threshold value, but the backward process does not occur. Such switches are indeed observed in computer simulations.

There is a growing interest in nonequilibrium dynamics of quasi-two-dimensional magnetic materials.[1–4] Many magnetic properties of these materials are well described by the classical two-dimensional Heisenberg model with easy-plane symmetry. In this model vortices play a very important role. They cause a topological phase transition,[5] and they contribute to the so-called central peaks in inelastic neutron scattering experiments[6–8] that arise from the translational motion of vortices.[9,10]

There are two types of static vortex solutions, depending on the anisotropy strength δ (see below) of the Heisenberg exchange interaction:[10] in-plane vortices for which all spins lie in the easy xy-plane, and out-of-plane vortices which exhibit a localized structure of the z-components of the spins around the vortex center. In addition to the vorticity $q=\pm 1,\pm 2,\ldots$, the out-of-plane vortices have a second topological charge p. This is denoted as ''polarization'' because its sign determines the side of the xy plane to which the out-of-plane vortex structure points.

The aim of the present paper is to investigate the internal dynamic of the vortices in the discrete two-dimensional Heisenberg ferromagnet in the presence of an ac magnetic field in the easy-plane. It is shown that switching between vortex states with different polarization occurs if the amplitude of the field is larger than a threshold value.

The easy-plane Hamiltonian for classical spins $\mathbf{S}_\mathbf{n}=S\{\sin\theta_\mathbf{n}\cos\Phi_\mathbf{n},\sin\theta_\mathbf{n}\sin\Phi_\mathbf{n},\cos\theta_\mathbf{n}\}$ located on sites $\mathbf{n}=(n_x,n_y)$ of a quadratic lattice has the form

$$H=-J\sum_{\mathbf{n},\mathbf{a}}\{(1-\delta)M_\mathbf{n}M_{\mathbf{n}-\mathbf{a}}+\sqrt{1-M_\mathbf{n}^2}\sqrt{1-M_{\mathbf{n}-\mathbf{a}}^2}\times\cos(\Phi_\mathbf{n}-\Phi_{\mathbf{n}-\mathbf{a}})\} \tag{1}$$

where δ is the anisotropy parameter $(0<\delta\leqslant 1)$, $J>0$ is the exchange constant and $\mathbf{a}$ is the vector which connects a site with nearest neighbors. $S_\mathbf{n}^z\equiv M_\mathbf{n}=\cos\theta_\mathbf{n}$ is the on-site magnetization. In what follows we set $J=1$ and $S=1$.

It is known[10,11] that for $\delta>\delta_c$ where the critical value of the anisotropy parameter δ_c depends on the lattice type(e.g. for square lattices $\delta_c\simeq 0.3$) the in-plane vortex with $M_\mathbf{n}=0$ and the azimuthal angles $\Phi_\mathbf{n}$ satisfying the equation $\Sigma_\mathbf{a}\sin(\Phi^0_\mathbf{n}-\Phi^0_{\mathbf{n}-\mathbf{a}})=0$, is stable. For $\delta<\delta_c$ the in-plane vortex becomes unstable and an out-of-plane vortex is created. It exhibits a localized structure of the $M_\mathbf{n}$ components around the vortex center. In the case of a circular system of the radius L with free boundary conditions the azimuthal angles $\Phi_\mathbf{n}$ for both types of vortices are approximately given by

$$\Phi_\mathbf{n}=q\arctan\left(\frac{n_y-Y}{n_x-X}\right)-q\arctan\left(\frac{n_y-\bar{Y}}{n_x-\bar{X}}\right), \tag{2}$$

where a constant phase has been omitted. X and Y are the coordinates of the vortex center, and $\bar{X}=XL^2/R^2$, $\bar{Y}=YL^2/R^2$ $(R^2=X^2+Y^2)$ are the coordinates of the ''image'' vortex. In the case of fixed boundary conditions the sign in front of the second term in Eq. (2) is reversed.

We are interested here in the vortex dynamics under the influence of a spatially uniform in-plane ac magnetic field $\mathbf{h}(t)=h(\cos\omega t,\sin\omega t,0)$. The interaction of the field with the spin system has the form

$$V(t)=-h\sum_\mathbf{n}\sqrt{1-M_\mathbf{n}^2}\cos(\Phi_\mathbf{n}-\omega t). \tag{3}$$

The spin dynamics is described by the Landau-Lifshitz equation

$$\begin{aligned}\dot{\Phi}_\mathbf{n}&=\frac{\partial}{\partial M_\mathbf{n}}[H+V(t)]-\frac{\gamma}{1-M_\mathbf{n}^2}\frac{\partial H}{\partial\Phi_\mathbf{n}},\\ \dot{M}_\mathbf{n}&=-\frac{\partial}{\partial\Phi_\mathbf{n}}[H+V(t)]-\gamma(1-M_\mathbf{n}^2)\frac{\partial H}{\partial M_\mathbf{n}}.\end{aligned} \tag{4}$$

The last terms in Eqs. (4) represent damping.[12]

To clarify the behavior of out-of-plane vortices in the presence of the ac field, we have numerically integrated the

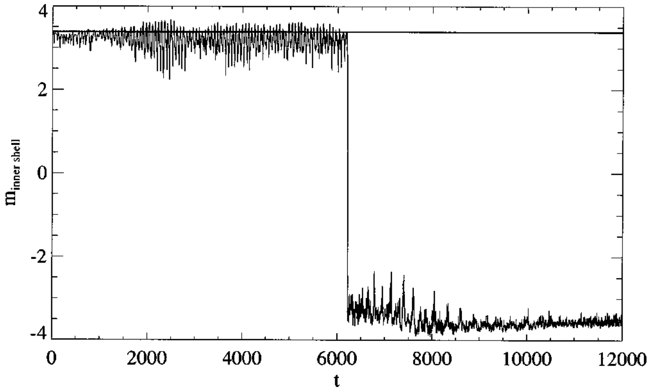

FIG. 1. Switching from the state with positive polarization to the state with negative polarization of the out-of-plane vortex due to a clockwise rotating magnetic field with the frequency $\omega=-0.1$. The damping constant $\gamma=0.002$. The lower curve shows the time evolution of the magnetization of the inner shell when the field amplitude $h=3\times10^{-3}$ is above the threshold value, the straight line corresponds to $h=10^{-3}$ (the amplitude of oscillations in this case is very small ≈ 0.005 and therefore they are not seen in the figure).

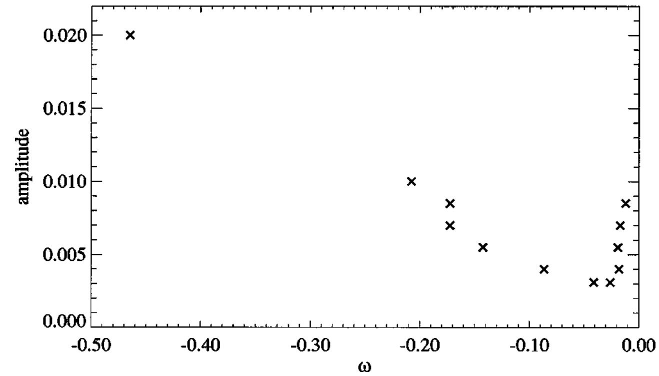

FIG. 2. Threshold value of the field amplitude h_{cr} versus ω obtained from the numerical integration of the full Landau-Lifshitz equations.

Landau-Lifshitz equation (4) for a large square lattice in which we cut out a circle with radius $L=24$ using both free and fixed boundary conditions and $\delta=0.1,\gamma=0.002$. We used relatively weak ac fields so as not to change the ground state significantly. The integration time was 12 000 time units with time step 0.01. First we used an out-of-plane vortex with polarization p=1 as the initial condition and a clockwise rotating magnetic field with the frequency $\omega=-0.1$: This is close to the frequency of the lowest radially symmetric eigenmode in the presence of a vortex.[11,13–15] We observed that for all $h\leqslant h_{cr}=0.0025$ the vortex with $p=1$ remains the stable configuration but for $h>h_{cr}$ a flip to the state with the opposite polarization ($p=-1$) occurs (Fig. 1). Using the same initial condition but changing the direction of rotation of the magnetic field ($\omega=0.1$) we observed the switching only when $h>0.02$. But in contrast to the previous case when the vortex after switching had a well-defined core structure, now the out-of-plane structure is almost completely destroyed by spin waves.

Another set of simulations was performed using a static out-of-plane vortex state with the same h but polarization $p=-1$ as initial condition. We found that the flip occurs only for a counterclockwise rotating magnetic field $\omega>0$. The results of extensive simulations may be summarized as follows for both types of boundary conditions:

(1) Flips between oppositely polarized states take place under the action of the ac magnetic field when $h>h_{cr}$ (Fig. 2).

(2) The threshold value h_{cr} depends on the product ωp and not on the vorticity. The threshold value in the case when the polarization vector is antiparallel to the angular velocity vector $\boldsymbol{\omega}=(0,0,\omega)$ is much smaller than when these vectors are parallel.

(3) Flips are unidirectional. When $\omega p<0$, the final state is characterized by a well-defined core structure, while for $\omega p>0$ the core structure is destroyed.

The basic reason for the switching can be easily understood by using the frame of reference which rotates together with the magnetic field. In this frame there exists an inertial force equivalent to a magnetic field aligned along the angular velocity $\boldsymbol{\omega}$. Then the vortex states with different polarization are nonequivalent and switching processes become energetically favored. This cannot explain, however, why the threshold of the switching is a nonmonotonic function of the frequency ω (Fig. 2). To gain deeper insight we need a reduced form of the Hamiltonian (1) which takes into account both types of vortices: in plane and out of plane. As a topological charge, p is conserved in the continuum limit only. Thus the switching between states with different polarization is due to lattice discreteness.

We consider the near-critical case $|(\delta-\delta_c)/(1-\delta_c)|\ll 1$ when the out-of-plane spin deviations $M_{\mathbf{n}}$ are small, and assume also smooth dependence of the deviations $\phi_{\mathbf{n}}=\Phi_{\mathbf{n}}-\Phi^0_{\mathbf{n}}$ from the static vortex structure on the spatial variable $\mathbf{n}$. In this case, applying the transformation

$$M_{\mathbf{n}}=\sum_{\nu}\mathcal{L}_{\mathbf{n}\nu}m_{\nu},\quad \phi_{\mathbf{n}}=\sum_{\nu}\mathcal{K}_{\mathbf{n}\nu}\psi_{\nu} \tag{5}$$

where the coefficients $\mathcal{L}_{\mathbf{n},\nu}$, $\mathcal{K}_{\mathbf{n},\nu}$ satisfy the set of equations $\mathcal{L}_{\mathbf{n},\nu}=2\Sigma_{\mathbf{a}}(\mathcal{K}_{\mathbf{n},\nu}-\mathcal{K}_{\mathbf{n}-\mathbf{a},\nu})\cos(\Phi^0_{\mathbf{n}}-\Phi^0_{\mathbf{n}-\mathbf{a}})$, $\mu_{\nu}\mathcal{K}_{\mathbf{n},\nu}=2\Sigma_{\mathbf{a}}[-(1-\delta)\mathcal{L}_{\mathbf{n}-\mathbf{a},\nu}+\mathcal{L}_{\mathbf{n},\nu}\cos(\Phi^0_{\mathbf{n}}-\Phi^0_{\mathbf{n}-\mathbf{a}})]$ we transform the harmonic part of the Hamiltonian (1) obtained in the vicinity of the static in-plane vortex,

$$H_0=\frac{1}{2}\sum_{\mathbf{n},\mathbf{a}}(\phi_{\mathbf{n}}-\phi_{\mathbf{n}-\mathbf{a}})^2\cos(\Phi^0_{\mathbf{n}}-\Phi^0_{\mathbf{n}-\mathbf{a}})-\sum_{\mathbf{n},\mathbf{a}}[(1-\delta)M_{\mathbf{n}}M_{\mathbf{n}-\mathbf{a}}-M^2_{\mathbf{n}}\cos(\Phi^0_{\mathbf{n}}-\Phi^0_{\mathbf{n}-\mathbf{a}})], \tag{6}$$

to the principal-axis coordinates $H_0=\frac{1}{2}\Sigma_{\nu}(\psi^2_{\nu}+\mu_{\nu}m^2_{\nu})$.

In Refs. 13 and 14 the linear eigenmodes of the easy-plane ferromagnet with Hamiltonian (1) in the presence of a vortex were investigated. The lowest radially symmetric mode, which is localized near the vortex center and describes the in-phase motion of the core spins, becomes soft when δ approaches δ_c. In other words, this mode is responsible for the in-plane vortex instability.

The corresponding eigenvalue, say μ_1, becomes negative when $\delta<\delta_c$: $\mu_1=B(\delta-\delta_c)$ with B a numerical coefficient. Inserting the transformation (5) into the Hamiltonian H_1

$=H-H_0$ and keeping only the soft eigenmode $\nu=1$, we obtain an effective soft-mode Hamiltonian

$$H_s=\frac{1}{2}(\psi_1^2+\mu_1 m_1^2)+\frac{A}{4}m_1^4, \tag{7}$$

where terms $m_1^2\psi_1^2$ and ψ_1^4 which are unimportant in the near-critical case and all higher order terms have been omitted. $A=\frac{1}{2}\Sigma_{\mathbf{n},\mathbf{a}}(\mathcal{L}_{\mathbf{n},1}^2-\mathcal{L}_{\mathbf{n}-\mathbf{a},1}^2)^2\cos(\Phi_{\mathbf{n}}^0-\Phi_{\mathbf{n}-\mathbf{a}}^0)$ is a positive constant.

We see from Eqs (3) and (2) that a spatially uniform in-plane magnetic field cannot excite the radially symmetric soft mode when the vortex is situated at the center of the system. Switching can occur only as a result of nonlinear mixing between the radially symmetric mode and nonsymmetric vortex modes which do interact with the spatially uniform alternating external field. This may take place in large systems where the motion of the vortex is frozen.[16] However, in a relatively small, finite system with free boundary conditions the vortex is attracted by its image and moves along an unwinding spiral trajectory (see e.g., Ref. 17) towards the boundary. Our numerical experiments do show that switching events in general occur only when the vortex is at a finite distance from the center. The vortex center motion is very slow (with a frequency $\sim 1/L^2$). So we can consider the switching process with fixed vortex position, say $X=R\cos\chi, Y=R\sin\chi$. Inserting Eqs. (5) and (2) into Eq. (3) and assuming that the vortex is far from the boundaries ($R \ll L$), we find that the effective interaction of the in-plane ac magnetic field with the soft mode is

$$V_s(t)=h\left(a_1\psi_1\sin(\omega t)-h\frac{1}{2}(a_2\psi_1^2+bm_1^2)\cos(\omega t)\right), \tag{8}$$

where $a_l=\Sigma_{\mathbf{n}}(R\sqrt{n_x^2+n_y^2}/2L^2)(\mathcal{K}_{\mathbf{n},1})^l (l=1,2)$, and $b=\Sigma_{\mathbf{n}}(R\sqrt{n_x^2+n_y^2}/2L^2)(\mathcal{L}_{\mathbf{n},1})^2$. An effective interaction of the same form can be obtained by taking into account the fact that the vortex structure is velocity dependent[10] and antisymmetric about the direction of the vortex motion. The constant phase shift ($q\chi$) plays no essential role and was omitted.

From Eqs. (7) and (8) we find that in the soft-mode approach the dynamics is governed by

$$\dot{m}_1=-\psi_1-h(a_1\sin\omega t-a_2\psi_1\cos\omega t)-\gamma(\mu_1 m_1+Am_1^3), \tag{9}$$

$$\dot{\psi}_1=\mu_1 m_1+Am_1^3-\gamma\psi_1-hbm_1\cos\omega t.$$

Here the nonlinear terms m_1^n ($n>3$) and $m_1^2\psi_1$ have been neglected. An example of the core dynamics based on Eqs. (9) is presented in Fig. 3. These results are in good agreement with those from the numerical integration of the full Landau-Lifshitz equations (4).

To clarify the physical meaning it is convenient to write the set of equations (9) as a single equation for m_1. Near the threshold ($|\mu_1|\ll 1$), in the limit of small damping ($\gamma\ll 1$), and for a not too strong amplitude of the external magnetic field ($a_2 h \ll 1$) we obtain from Eqs. (9) an effective equation for m_1: $\ddot{m}_1+\gamma\dot{m}_1+\mu_1 m_1+Am_1^3+h(a_1\omega-bm_1)\cos\omega t=0$. Thus the vortex core dynamics is analogous to the dynamics of a particle in a double-well potential under the action of direct and parametric forces.In order to estimate h_{cr} we use a heuristic approach proposed by Moon.[18] Here the switching occurs when the particle reaches the maximum velocity on the homoclinic orbit. Considering the particle motion in the potential well centered at $m_1=p\sqrt{\mu_1}$ and using multiple-scale analysis for small γ, h and $|\omega^2-\omega_0^2|\ll 1$, where $\omega_0=\sqrt{2|\mu_1|}$ is the frequency of harmonic oscillations near the bottom of the well, we find $m_1\approx\sqrt{|\mu_1|}[p+M(\omega)\cos(\omega t)]$, with

$$M^2\left[\left(\omega_0^2-\omega^2-\frac{3}{2}\omega_0^2 M^2\right)^2+\gamma^2\omega^2\right]=2A\frac{h^2}{\omega_0^4}(a_1\omega-b\sqrt{|\mu_1|}p)^2. \tag{10}$$

The maximum velocity on the homoclinic orbit is $|\mu_1|/4A$. The switching occurs when $\omega M(\omega)=\alpha|\mu_1|/4A$, where $\alpha\approx 1$ is an empirical parameter.From Eq. (10) we find $h_{cr}(\omega)$ in the form

$$\frac{\alpha\omega_0^2}{\sqrt{2A}}\frac{1}{|\Omega(a_1\sqrt{2}\Omega-bp)|}\sqrt{\left(1-\Omega^2-\frac{3}{8}\frac{\alpha^2}{\Omega^2}\right)^2+\frac{\gamma^2}{\omega_0^2}\Omega^2} \tag{11}$$

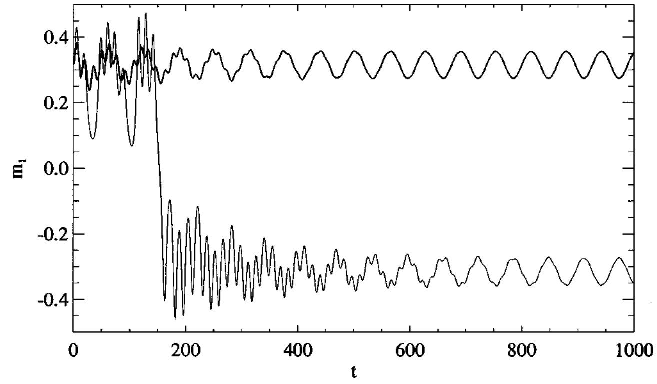

FIG. 3. Time evolution of the vortex core magnetization in the presence of the clockwise (upper curve) and counterclockwise (lower curve) rotating magnetic field based on Eqs (9). The parameters are $A=a_1=a_2=b=1$, $\mu_1=-0.1$, $\omega=\pm 0.1$, $h=0.0355$, $\gamma=0.01$. Initial polarization $p=+1$.

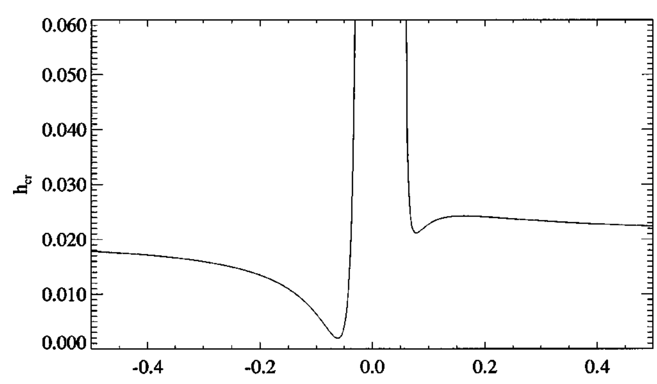

FIG. 4. Threshold value of the field amplitude h_{cr} versus ω, as given by Eq. (11). The parameters used are $a_1=b$, $Ab/\alpha=0.01$, $\omega_0=0.08$, $\gamma=0.002$.

where $\Omega=\omega/\omega_0$. The condition (11) is in agreement with the results of numerical simulations: the function $h_{cr}(\omega)$ has minima near the frequency of the soft mode $\omega\approx\pm\omega_0$. It is highly asymmetric (Fig. 4). For small amplitudes of the counterclockwise rotating magnetic field $(\omega>0)$, the switching condition $h>h_{cr}$ can be fulfilled only for the vortex with initial polarization $p=-1$, while for a clockwise rotating field the condition can be fulfilled for the vortex with opposite polarization $p=1$. Qualitatively the same results may be obtained by using the Melnikov function approach.[19] Our core model gives a rather good *qualitative* agreement with the results of numerical simulations in the case of alternating magnetic fields with low frequency. For high frequencies one should modify our approach by taking into account the change of the vortex structure in the presence of a rapidly rotating magnetic field in the easy plane.

In conclusion, we have shown that the polarization of out-of-plane vortices in easy-plane ferromagnets can be changed by applying an ac magnetic field. Flips occur much more easily when the polarization of the vortex is antiparallel to the angular velocity $\boldsymbol{\omega}$. Flips can take place only in discrete systems, and they are unidirectional events.

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